Math 1A (7:30am – 8:20am)
Quiz 3 Version B
Fri Oct 8, 2010

NAME YOU ASKED TO BE CALLED: _____

SCORE: ___ / 30 POINTS

NO CALCULATORS ALLOWED
SHOW PROPER CALCULUS-LEVEL ALGEBRAIC WORK
USE PROPER NOTATION

Prove that the equation $x^4 = x+1$ has a solution in the interval $(-1, 1)$. Justify your reasoning properly. **SCORE: ___ / 5 POINTS**

LET $f(x) = x^4 - x - 1$

f IS CONT. ON $[-1,1]$ SINCE f IS A POLYNOMIAL

$f(-1) = 1$ AND $f(1) = -1$, SO $f(1) < 0 < f(-1)$

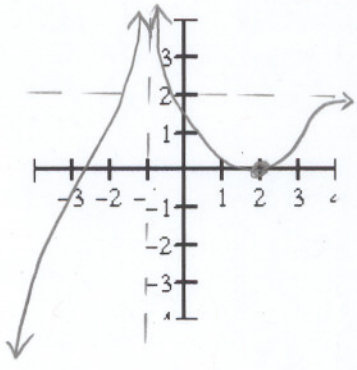
BY IVT, THERE IS A $c \in (-1,1)$ SUCH THAT $f(c) = 0$

IE. $c^4 - c - 1 = 0$

$c^4 = c + 1$

Sketch the graph of a function f such that **SCORE: ___ / 3 POINTS**

f is continuous everywhere except at $x = -1$,
$f(2) = 0$,
$\lim_{x \to -1} f(x) = \infty$,
$\lim_{x \to -\infty} f(x) = -\infty$ and
$\lim_{x \to \infty} f(x) = 2$.



Find $\lim_{x \to -\infty} \dfrac{5+4x}{\sqrt{7+9x^2}}$. **SCORE: ___ / 5 POINTS**

$$= \lim_{x \to -\infty} \frac{5+4x}{\sqrt{7+9x^2}} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}$$

$$= \lim_{x \to -\infty} \frac{\frac{5}{x}+4}{\sqrt{7+9x^2} \cdot -\sqrt{\frac{1}{x^2}}}$$

$$= \lim_{x \to -\infty} \frac{\frac{5}{x}+4}{-\sqrt{\frac{7}{x^2}+9}}$$

$$= \frac{0+4}{-\sqrt{0+9}} = -\frac{4}{3}$$

State the definition of "horizontal asymptote". Write in complete sentences, using proper English and algebra. **SCORE: ___ / 2 POINTS**

f HAS A HORIZONTAL ASYMPTOTE AT $y = b$

IF $\lim_{x \to \infty} f(x) = b$ OR $\lim_{x \to -\infty} f(x) = b$

CONTINUED ON OTHER SIDE

The time it takes to run a certain distance depends on the speed you run. If $t = f(v)$, where t is the time (in minutes), and v is your speed (in feet per second), what does the statement $f'(8) = -6$ mean ?
Give the units of measurement for each number in your answer.
NOTE: Your answer should NOT include "derivative", "instantaneous", "rate of change", "with respect to", "slope" or "tangent line".

SCORE: ___ / 2 POINTS

IF YOU RUN AT 8 FT/SEC,
YOUR TIME WILL DECREASE BY 6 MINUTES
FOR EACH FT/SEC YOU RUN FASTER

Let $f(x) = -\frac{1}{x^2}$.

SCORE: ___ / 8 POINTS

[a] Find $f'(x)$. **NOTE: If you have taken calculus before, do NOT use differentiation shortcuts.**

SEE VERSION A KEY

[b] Find the **equation** of the tangent line to the graph of $y = f(x)$ at $x = 2$.

$f'(2) = \frac{1}{4}$

$y - -\frac{1}{4} = \frac{1}{4}(x-2)$

$y + \frac{1}{4} = \frac{1}{4}(x-2)$ OR $y = -\frac{1}{4} + \frac{1}{4}(x-2)$ OR $y = \frac{1}{4}x - \frac{3}{4}$

State the Squeeze Theorem. Write in complete sentences, using proper English and algebra.

SCORE: ___ / 2 POINTS

IF $f(x) \le g(x) \le h(x)$ FOR ALL x NEAR a (EXCEPT POSSIBLY AT a)
AND $\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L$, THEN $\lim_{x\to a} g(x) = L$

Fill in each blank with either a number, ∞, $-\infty$ or DNE (write DNE if no other answer is appropriate).
NOTE: You do NOT need to show work.

SCORE: ___ / 3 POINTS

[a] $\lim_{x\to\infty} \arctan x = \frac{\pi}{2}$

[b] $\lim_{x\to-\infty} (1 - x^3) = \infty$

[c] $\lim_{x\to\infty} 3^{-x} = 0$