SCORE: ___ / 30 POINTS

NO CALCULATORS ALLOWED SHOW PROPER CALCULUS-LEVEL ALGEBRAIC WORK USE PROPER NOTATION

State the Intermediate Value Theorem. Write in complete sentences, using proper English and algebra.

SCORE: ___/3 POINTS

SCORE: ___/3 POINTS

Fill in each blank with either a number, ∞ , $-\infty$ or DNE (write DNE if no other answer is appropriate). **NOTE: You do NOT need to show work.**

$$\lim_{x \to -\infty} (1 - x^3) = \infty$$

[b]
$$\lim_{x \to \infty} (0.3)^x = \bigcirc$$

[c]
$$\lim_{x \to -\infty} \arctan x = -\frac{\pi}{2}$$

If $1+2x-x^2 \le f(x) \le x^4-2x^2+3$ for all $x \in (-2,3)$, find $\lim_{x\to 1} f(x)$. Justify your answer properly.

SCORE: / 4 POINTS

$$\lim_{x \to 1} (1+2x-x^2) = 2$$

$$\lim_{x \to 1} (x^4-2x^2+3) = 2$$

The number of cars sold by a dealer each week depends on the money it spends on advertising. If c = f(a), SCORE: ___/2 POINTS where c is the number of cars, and a is the advertising spending (in thousands of dollars), what does the statement f'(7) = 2 mean? Give the units of measurement for each number in your answer.

NOTE: Your answer should NOT include "derivative", "instantaneous", "rate of change", "with respect to", "slope" or "tangent line".

1F THE DEALER SPENDS \$7000 ON ADVERTISING,
2 MORE CARS WILL BE SOLD EACH WEEK
FOR EVERLY ADDITIONAL \$ 1000 SPENT ON ADVERTISING.

State the definition of "horizontal asymptote". Write in complete sentences, using proper English and algebra.

SCORE: / 2 POINTS

Let
$$f(x) = \frac{1}{x^2}$$
.

SCORE: ___/8 POINTS

[a] Find f'(x). NOTE: If you have taken calculus before, do NOT use differentiation shortcuts.

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - \frac{1}{x^2}}{h}$$

$$= \lim_{h \to 0} \frac{x^2 - (x+h)^2}{h(x+h)^2 x^2}$$

$$= \lim_{h \to 0} \frac{-2xh - h^2}{h(x+h)^2 x^2}$$

$$= \lim_{h \to 0} \frac{-2x - h}{(x+h)^2 x^2}$$

$$= -\frac{2}{x^3}$$

[b] Find the equation of the tangent line to the graph of y = f(x) at x = -3.

$$f'(-3) = \frac{2}{5}$$

 $y - \dot{q} = \frac{2}{5}(x - 3)$
 $y - \dot{q} = \frac{2}{5}(x + 3)$ or $y = \dot{q} + \frac{2}{5}(x + 3)$ or $y = \frac{2}{5}x + \frac{1}{3}$

Find
$$\lim_{x \to -\infty} \frac{\sqrt{5 + x^2}}{7 + 4x}$$
.

SCORE: ___ / 5 POINTS

$$x \to -\infty \qquad 7 + 4x$$

$$= \lim_{x \to -\infty} \frac{\sqrt{5 + x^2}}{\sqrt{7 + 4x}} \cdot \frac{1}{x}$$

$$= \lim_{x \to -\infty} \frac{\sqrt{5 + x^2}}{\sqrt{7 + 4x}} \cdot \frac{1}{x}$$

$$= \lim_{x \to -\infty} -\sqrt{\frac{5}{x^2 + 1}}$$

$$= \lim_{x \to -\infty} -\sqrt{\frac{5}{x^2 + 1}} \cdot \frac{1}{x}$$
Sketch the graph of a function f such that

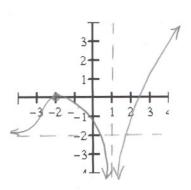
f is continuous everywhere except at x = 1,

$$f(-2)=0\,,$$

$$\lim_{x \to 1} f(x) = -\infty,$$

$$\lim_{x \to \infty} f(x) = \infty$$
 and

$$\lim f(x) = -2.$$



SCORE: ___/ 3 POINTS