Math 1A (7:30am – 8:20am)
Quiz 3 Version C
Fri Oct 8, 2010

NAME YOU ASKED TO BE CALLED: _____

SCORE: ___ / 30 POINTS

NO CALCULATORS ALLOWED
SHOW PROPER CALCULUS-LEVEL ALGEBRAIC WORK
USE PROPER NOTATION

State the Intermediate Value Theorem. Write in complete sentences, using proper English and algebra. **SCORE: ___ / 3 POINTS**

IF f IS CONTINUOUS ON $[a,b]$ AND d IS BETWEEN $f(a)$ AND $f(b)$, THEN THERE IS A $c \in (a,b)$ SUCH THAT $f(c)=d$

Fill in each blank with either a number, ∞, $-\infty$ or DNE (write DNE if no other answer is appropriate). **SCORE: ___ / 3 POINTS**
NOTE: You do NOT need to show work.

[a] $\lim_{x \to -\infty} (1 - x^3) =$ ∞

[b] $\lim_{x \to \infty} (0.3)^x =$ 0

[c] $\lim_{x \to -\infty} \arctan x =$ $-\frac{\pi}{2}$

If $1 + 2x - x^2 \le f(x) \le x^4 - 2x^2 + 3$ for all $x \in (-2, 3)$, find $\lim_{x \to 1} f(x)$. Justify your answer properly. **SCORE: ___ / 4 POINTS**

$\lim_{x \to 1} (1 + 2x - x^2) = 2$

$\lim_{x \to 1} (x^4 - 2x^2 + 3) = 2$

BY SQUEEZE THEOREM, $\lim_{x \to 1} f(x) = 2$

The number of cars sold by a dealer each week depends on the money it spends on advertising. If $c = f(a)$, where c is the number of cars, and a is the advertising spending (in thousands of dollars), what does the statement $f'(7) = 2$ mean ? Give the units of measurement for each number in your answer. **SCORE: ___ / 2 POINTS**
NOTE: Your answer should NOT include "derivative", "instantaneous", "rate of change", "with respect to", "slope" or "tangent line".

IF THE DEALER SPENDS \$7000 ON ADVERTISING,
2 MORE CARS WILL BE SOLD EACH WEEK
FOR EVERY ADDITIONAL \$1000 SPENT ON ADVERTISING.

State the definition of "horizontal asymptote". Write in complete sentences, using proper English and algebra. **SCORE: ___ / 2 POINTS**

f HAS A HORIZONTAL ASYMPTOTE AT $y=b$
IF $\lim_{x \to \infty} f(x) = b$ OR $\lim_{x \to -\infty} f(x) = b$

CONTINUED ON OTHER SIDE

Let $f(x)=\frac{1}{x^2}$.

SCORE: ___ / 8 POINTS

[a] Find $f'(x)$. **NOTE: If you have taken calculus before, do NOT use differentiation shortcuts.**

$$f'(x)=\lim_{h\to 0}\frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h}$$

$$=\lim_{h\to 0}\frac{x^2-(x+h)^2}{h(x+h)^2x^2}$$

$$=\lim_{h\to 0}\frac{-2xh-h^2}{h(x+h)^2x^2}$$

$$=\lim_{h\to 0}\frac{-2x-h}{(x+h)^2x^2}$$

$$=\frac{-2x}{x^4}$$

$$=-\frac{2}{x^3}$$

[b] Find the **equation** of the tangent line to the graph of $y=f(x)$ at $x=-3$.

$$f'(-3)=\frac{2}{27}$$

$$y-\frac{1}{9}=\frac{2}{27}(x-^-3)$$

$$y-\frac{1}{9}=\frac{2}{27}(x+3) \text{ OR } y=\frac{1}{9}+\frac{2}{27}(x+3) \text{ OR } y=\frac{2}{27}x+\frac{1}{3}$$

Find $\lim_{x\to-\infty}\frac{\sqrt{5+x^2}}{7+4x}$.

SCORE: ___ / 5 POINTS

$$=\lim_{x\to-\infty}\frac{\sqrt{5+x^2}}{7+4x}\cdot\frac{\frac{1}{x}}{\frac{1}{x}}$$

$$=\lim_{x\to-\infty}\frac{\sqrt{5+x^2}\cdot-\sqrt{\frac{1}{x^2}}}{\frac{7}{x}+4}$$

$$=\lim_{x\to-\infty}\frac{-\sqrt{\frac{5}{x^2}+1}}{\frac{7}{x}+4}$$

$$=\frac{-\sqrt{0+1}}{0+4}=\frac{-1}{4}$$

Sketch the graph of a function f such that

SCORE: ___ / 3 POINTS

f is continuous everywhere except at $x=1$,
$f(-2)=0$,
$\lim_{x\to 1}f(x)=-\infty$,
$\lim_{x\to\infty}f(x)=\infty$ and
$\lim_{x\to-\infty}f(x)=-2$.