Math 1A (7:30am – 8:20am)
Quiz 3 Version D
Fri Oct 8, 2010

NAME YOU ASKED TO BE CALLED: _____

SCORE: ___ / 30 POINTS

NO CALCULATORS ALLOWED
SHOW PROPER CALCULUS-LEVEL ALGEBRAIC WORK
USE PROPER NOTATION

Let $f(x) = \dfrac{1}{x^2}$.

SCORE: ___ / 8 POINTS

[a] Find $f'(x)$. **NOTE: If you have taken calculus before, do NOT use differentiation shortcuts.**

SEE VERSION C KEY

[b] Find the **equation** of the tangent line to the graph of $y = f(x)$ at $x = -2$.

$f'(-2) = \frac{1}{4}$

$y - \frac{1}{4} = \frac{1}{4}(x - -2)$

$y - \frac{1}{4} = \frac{1}{4}(x+2)$ OR $y = \frac{1}{4} + \frac{1}{4}(x+2)$ OR $y = \frac{1}{4}x + \frac{3}{4}$

Find $\lim\limits_{x \to -\infty} \dfrac{\sqrt{4+9x^2}}{5+2x}$.

SCORE: ___ / 5 POINTS

$= \lim\limits_{x \to -\infty} \dfrac{\sqrt{4+9x^2}}{5+2x} \cdot \dfrac{\frac{1}{x}}{\frac{1}{x}}$

$= \lim\limits_{x \to -\infty} \dfrac{\sqrt{4+9x^2} \cdot -\sqrt{\frac{1}{x^2}}}{\frac{5}{x}+2}$

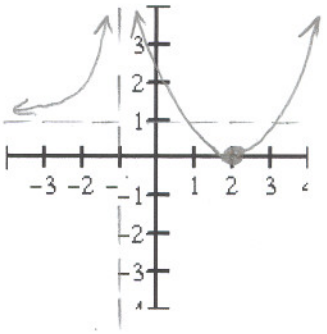
$= \lim\limits_{x \to -\infty} \dfrac{-\sqrt{\frac{4}{x^2}+9}}{\frac{5}{x}+2}$

$= \dfrac{-\sqrt{0+9}}{0+2} = -\dfrac{3}{2}$

CONTINUED ON OTHER SIDE

Sketch the graph of a function f such that **SCORE: ___ / 3 POINTS**

f is continuous everywhere except at $x = -1$,
$f(2) = 0$,
$\lim_{x \to -1} f(x) = \infty$,
$\lim_{x \to \infty} f(x) = \infty$ and
$\lim_{x \to -\infty} f(x) = 2$.



If $3 + 2x - x^2 \le f(x) \le x^4 - 2x^2 + 5$ for all $x \in (-1, 5)$, find $\lim_{x \to 1} f(x)$. Justify your answer properly. **SCORE: ___ / 4 POINTS**

$\lim_{x \to 1} (3 + 2x - x^2) = 4$

$\lim_{x \to 1} (x^4 - 2x^2 + 5) = 4$

BY SQUEEZE THEOREM, $\lim_{x \to 1} f(x) = 4$

The number of TVs sold by a store each week depends on the money it spends on sales staff bonuses. If $t = f(b)$, **SCORE: ___ / 2 POINTS**
where t is the number of TVs, and b is the amount spent on bonuses (in thousands of dollars), what does the statement $f'(11) = 4$ mean ?
Give the units of measurement for each number in your answer.
NOTE: Your answer should NOT include "derivative", "instantaneous", "rate of change", "with respect to", "slope" or "tangent line".

IF \$11,000 IS GIVEN OUT AS BONUSES,
WEEKLY SALES WOULD INCREASE BY 4 TV'S
FOR EACH ADDITIONAL \$1,000 GIVEN OUT AS BONUSES

Fill in each blank with either a number, ∞, $-\infty$ or DNE (write DNE if no other answer is appropriate). **SCORE: ___ / 3 POINTS**
NOTE: You do NOT need to show work.

[a] $\lim_{x \to -\infty} (1 - x^3) =$ ∞

[b] $\lim_{x \to -\infty} \arctan x =$ $-\frac{\pi}{2}$

[c] $\lim_{x \to -\infty} (0.3)^x =$ ∞

State the definition of "horizontal asymptote". Write in complete sentences, using proper English and algebra. **SCORE: ___ / 2 POINTS**

f HAS A HORIZONTAL ASYMPTOTE AT $y = b$
IF $\lim_{x \to \infty} f(x) = b$ OR $\lim_{x \to -\infty} f(x) = b$

State the Intermediate Value Theorem. Write in complete sentences, using proper English and algebra. **SCORE: ___ / 3 POINTS**

IF f IS CONTINUOUS ON $[a,b]$ AND d IS BETWEEN $f(a)$ AND $f(b)$,
THEN THERE IS A $c \in (a,b)$ SUCH THAT $f(c) = d$.