Math 1A (7:30am – 8:20am)
Quiz 4 Version A
Fri Oct 22, 2010

NAME YOU ASKED TO BE CALLED: ______________

SCORE: ___ / 30 POINTS

NO CALCULATORS ALLOWED
SHOW PROPER WORK / USE PROPER NOTATION / SIMPLIFY YOUR ANSWERS

If $f(x) = \dfrac{5x^3 + 3x^2 - 3}{\sqrt{x}}$, find $f''(x)$.

SCORE: ___ / 3 POINTS

$f(x) = 5x^{\frac{5}{2}} + 3x^{\frac{3}{2}} - 3x^{-\frac{1}{2}}$

$f'(x) = \frac{25}{2}x^{\frac{3}{2}} + \frac{9}{2}x^{\frac{1}{2}} + \frac{3}{2}x^{-\frac{3}{2}}$

$f''(x) = \frac{75}{4}x^{\frac{1}{2}} + \frac{9}{4}x^{-\frac{1}{2}} - \frac{9}{4}x^{-\frac{5}{2}}$

If $f(x) = \sin x$, find $f^{(31)}(x)$. **NOTE: You do not need to show all 30 derivatives before the 31^{st} derivative, but you should show how you got your answer.**

SCORE: ___ / 3 POINTS

$f'(x) = \cos x = f^{(5)}(x)$

$f''(x) = -\sin x = f^{(6)}(x)$

$f'''(x) = -\cos x = f^{(7)}(x) \quad = f^{(31)}(x) \qquad f^{(31)}(x) = -\cos x$

$f^{(4)}(x) = \sin x = f^{(8)}(x) \ldots = f^{(32)}(x)$

The fuel efficiency of your car depends on the speed at which you drive. If $E = f(v)$, where E is the fuel efficiency (in miles per gallon), and v is your speed (in miles per hour), what does the statement $f'(67) = -0.2$ mean? Give the units of measurement for each number in your answer.

SCORE: ___ / 3 POINTS

NOTE: Your answer should NOT include "derivative", "instantaneous", "rate of change", "with respect to", "slope" or "tangent line".

IF YOU DRIVE 67 MPH,
YOUR FUEL EFFICIENCY DROPS 0.2 MPG
FOR EACH MPH YOU DRIVE FASTER

Prove that if $f(x) = \cos x$, then $f'(x) = -\sin x$ using the definition of the derivative. You may use the two limits proved in class without proving them again.

SCORE: ___ / 3 POINTS

$f'(x) = \lim_{h \to 0} \dfrac{\cos(x+h) - \cos x}{h}$

$= \lim_{h \to 0} \dfrac{\cos x \cosh - \sin x \sin h - \cos x}{h}$

$= \lim_{h \to 0} \left[\cos x \left(\dfrac{\cos h - 1}{h}\right) - \sin x \left(\dfrac{\sin h}{h}\right)\right]$

$= (\cos x) \cdot 0 - (\sin x) \cdot 1$

$= -\sin x$

If $f(x) = \dfrac{\csc x}{1 + \tan x}$, find $f'(x)$. **SCORE: ___ / 4 POINTS**

$$f'(x) = \frac{(-\csc x \cot x)(1+\tan x) - \csc x \sec^2 x}{(1+\tan x)^2}$$

$$= \frac{-\csc x(\cot x + 1 + \sec^2 x)}{(1+\tan x)^2}$$

If $f(x) = \sqrt[3]{x} \cot x$, find $f'(x)$. **SCORE: ___ / 4 POINTS**

$$f'(x) = \tfrac{1}{3}x^{-\frac{2}{3}}\cot x - x^{\frac{1}{3}}\csc^2 x$$

$$= \tfrac{1}{3}x^{-\frac{2}{3}}(\cot x - 3x\csc^2 x)$$

Let $y = \dfrac{x^2 - x}{2x + 1}$. **SCORE: ___ / 6 POINTS**

[a] Find $\left.\dfrac{dy}{dx}\right|_{x=2}$.

$$\frac{dy}{dx} = \frac{(2x-1)(2x+1) - 2(x^2-x)}{(2x+1)^2}$$

$$\left.\frac{dy}{dx}\right|_{x=2} = \frac{(3)(5) - 2(2)}{5^2} = \frac{11}{25}$$

[b] Find the equation of the normal line at $x = 2$.

$m = -\frac{25}{11}$

WHEN $x = 2,\ y = \frac{2}{5}$

$$y - \tfrac{2}{5} = -\tfrac{25}{11}(x-2)$$

The table below shows values of $f(x)$ and $f'(x)$ for several values of x. **SCORE: ___ / 4 POINTS**

If $g(x) = x^3 f(x)$, find $g'(-2)$.

x	-3	-2	-1	0	1	2	3
$f(x)$	2	-1	-3	-2	3	1	0
$f'(x)$	-1	3	0	-2	-3	-1	2

$$g'(x) = 3x^2 f(x) + x^3 f'(x)$$

$$g'(-2) = 3(-2)^2 f(-2) + (-2)^3 f'(-2)$$

$$= 3(4)(-1) + (-8)(3)$$

$$= -36$$