Math 1A (7:30am – 8:20am)
Quiz 4 Version C
Fri Oct 22, 2010

NAME YOU ASKED TO BE CALLED: _____

SCORE: ___ / 30 POINTS

NO CALCULATORS ALLOWED
SHOW PROPER WORK / USE PROPER NOTATION / SIMPLIFY YOUR ANSWERS

The time for a fire to burn out depends on the amount of wood being burned. If $t = f(w)$, where t is the time it takes the fire to burn out (in minutes), and w is the amount of wood (in pounds), what does the statement $f'(13) = 10$ mean? Give the units of measurement for each number in your answer.
NOTE: Your answer should NOT include "derivative", "instantaneous", "rate of change", "with respect to", "slope" or "tangent line".

SCORE: ___ / 3 POINTS

IF YOU BURN 13 POUNDS OF WOOD,
THE FIRE WILL BURN 10 MINUTES LONGER
FOR EACH POUND OF WOOD YOU ADD TO THE FIRE

Prove that if $f(x) = \cos x$, then $f'(x) = -\sin x$ using the definition of the derivative. You may use the two limits proved in class without proving them again.

SCORE: ___ / 3 POINTS

SEE VERSION A KEY

If $f(x) = \dfrac{3x^4 + 4x^2 - 2}{\sqrt{x}}$, find $f''(x)$.

SCORE: ___ / 3 POINTS

$$f(x) = 3x^{\frac{7}{2}} + 4x^{\frac{3}{2}} - 2x^{-\frac{1}{2}}$$
$$f'(x) = \frac{21}{2}x^{\frac{5}{2}} + 6x^{\frac{1}{2}} + x^{-\frac{3}{2}}$$
$$f''(x) = \frac{105}{4}x^{\frac{3}{2}} + 3x^{-\frac{1}{2}} - \frac{3}{2}x^{-\frac{5}{2}}$$

If $f(x) = \cos x$, find $f^{(31)}(x)$. **NOTE: You do not need to show all 30 derivatives before the 31^{st} derivative, but you should show how you got your answer.**

SCORE: ___ / 3 POINTS

$$f'(x) = -\sin x = f^{(5)}(x)$$
$$f''(x) = -\cos x = f^{(6)}(x)$$
$$f'''(x) = \sin x = f^{(7)}(x) = f^{(31)}(x)$$
$$f^{(4)}(x) = \cos x = f^{(8)}(x) = \dots = f^{(32)}(x)$$

$f^{(31)}(x) = \sin x$

Let $y = \dfrac{x^2 + x}{2x - 1}$.

SCORE: ___ / 6 POINTS

[a] Find $\left.\dfrac{dy}{dx}\right|_{x=3}$.

$$\frac{dy}{dx} = \frac{(2x+1)(2x-1) - 2(x^2+x)}{(2x-1)^2}$$

$$\left.\frac{dy}{dx}\right|_{x=3} = \frac{(7)(5) - 2(12)}{5^2} = \frac{11}{25}$$

[b] Find the equation of the normal line at $x = 3$.

$m = -\frac{25}{11}$

WHEN $x = 3$, $y = \frac{12}{5}$

$$y - \frac{12}{5} = -\frac{25}{11}(x - 3)$$

The table below shows values of $f(x)$ and $f'(x)$ for several values of x.

SCORE: ___ / 4 POINTS

If $g(x) = x^2 f(x)$, find $g'(-3)$.

x	–3	–2	–1	0	1	2	3
$f(x)$	2	–1	–3	–2	3	1	0
$f'(x)$	–1	3	0	–2	–3	–1	2

$g'(x) = 2x f(x) + x^2 f'(x)$

$g'(-3) = 2(-3) f(-3) + (-3)^2 f'(-3)$

$= (-6)(2) + (9)(-1)$

$= -21$

If $f(x) = \dfrac{\tan x}{1 + \csc x}$, find $f'(x)$.

SCORE: ___ / 4 POINTS

$$f'(x) = \frac{\sec^2 x (1 + \csc x) - \tan x (-\csc x \cot x)}{(1 + \csc x)^2}$$

$$= \frac{\sec^2 x + \sec^2 x \csc x + \csc x}{(1 + \csc x)^2}$$

If $f(x) = \sqrt[3]{x} \cot x$, find $f'(x)$.

SCORE: ___ / 4 POINTS

SEE VERSION A KEY