Math 1A (7:30am – 8:20am)
Quiz 4 Version D
Fri Oct 22, 2010

NAME YOU ASKED TO BE CALLED: _____

SCORE: ___ / 30 POINTS

NO CALCULATORS ALLOWED
SHOW PROPER WORK / USE PROPER NOTATION / SIMPLIFY YOUR ANSWERS

The height of a wine barrel depends on the area of its base. If $h = f(a)$, where h is the height (in inches), and a is the area of the base (in square inches), what does the statement $f'(150) = -0.2$ mean ? Give the units of measurement for each number in your answer.

SCORE: ___ / 3 POINTS

NOTE: Your answer should NOT include "derivative", "instantaneous", "rate of change", "with respect to", "slope" or "tangent line".

IF THE BASE AREA OF A WINE BARREL IS 150 INCHES^2, THE HEIGHT DECREASES BY 0.2 INCHES FOR EACH INCH^2 INCREASE IN THE AREA OF THE BASE

Prove that if $f(x) = \cos x$, then $f'(x) = -\sin x$ using the definition of the derivative. You may use the two limits proved in class without proving them again.

SCORE: ___ / 3 POINTS

SEE VERSION A KEY

If $f(x) = \dfrac{3x^3 + 5x^2 + 3}{\sqrt{x}}$, find $f''(x)$.

SCORE: ___ / 3 POINTS

$f(x) = 3x^{\frac{5}{2}} + 5x^{\frac{3}{2}} + 3x^{-\frac{1}{2}}$

$f'(x) = \frac{15}{2}x^{\frac{3}{2}} + \frac{15}{2}x^{\frac{1}{2}} - \frac{3}{2}x^{-\frac{3}{2}}$

$f''(x) = \frac{45}{4}x^{\frac{1}{2}} + \frac{15}{4}x^{-\frac{1}{2}} + \frac{9}{4}x^{-\frac{5}{2}}$

If $f(x) = \cos x$, find $f^{(29)}(x)$. **NOTE: You do not need to show all 28 derivatives before the 29th derivative, but you should show how you got your answer.**

SCORE: ___ / 3 POINTS

$f'(x) = -\sin x = f^{(5)}(x) \quad \cdots \quad = f^{(29)}(x)$

$f''(x) = -\cos x = f^{(6)}(x)$

$f'''(x) = \sin x = f^{(7)}(x)$

$f^{(4)}(x) = \cos x = f^{(8)}(x) = \cdots = f^{(28)}(x)$

$f^{(29)}(x) = -\sin x$

Let $y = \dfrac{2x-1}{x^2+x}$.

SCORE: ___ / 6 POINTS

[a] Find $\left.\dfrac{dy}{dx}\right|_{x=3}$.

$$\frac{dy}{dx} = \frac{2(x^2+x)-(2x-1)(2x+1)}{(x^2+x)^2}$$

$$\left.\frac{dy}{dx}\right|_{x=3} = \frac{2(12)-(5)(7)}{12^2} = \frac{-11}{144}$$

[b] Find the equation of the normal line at $x = 3$.

$m = \frac{144}{11}$

WHEN $x=3$, $y = \frac{5}{12}$

$$y - \frac{5}{12} = \frac{144}{11}(x-3)$$

The table below shows values of $f(x)$ and $f'(x)$ for several values of x.

SCORE: ___ / 4 POINTS

If $g(x) = x^2 f(x)$, find $g'(3)$.

x	–3	–2	–1	0	1	2	3
$f(x)$	2	–1	–3	–2	3	1	–3
$f'(x)$	–1	3	0	–2	–3	–1	–2

$$g'(x) = 2x f(x) + x^2 f'(x)$$

$$g'(3) = 2(3)f(3) + (3)^2 f'(3)$$

$$= (6)(-3) + (9)(-2)$$

$$= -36$$

If $f(x) = \dfrac{\cot x}{1+\sec x}$, find $f'(x)$.

SCORE: ___ / 4 POINTS

$$f'(x) = \frac{-\csc^2 x(1+\sec x) - \cot x(\sec x \tan x)}{(1+\sec x)^2}$$

$$= \frac{-\csc^2 x - \csc^2 x \sec - \sec x}{(1+\sec x)^2}$$

If $f(x) = \sqrt[3]{x}\csc x$, find $f'(x)$.

SCORE: ___ / 4 POINTS

SEE VERSION B KEY