Math 1A (7:30am – 8:20am)
Quiz 6 Version A
Thu Nov 11, 2010

NAME YOU ASKED TO BE CALLED: ____________________

SCORE: ___ / 30 POINTS

NO CALCULATORS ALLOWED
SHOW PROPER WORK / USE PROPER NOTATION / SIMPLIFY YOUR ANSWERS

State the definition of "critical number". **SCORE: ___ / 2 POINTS**

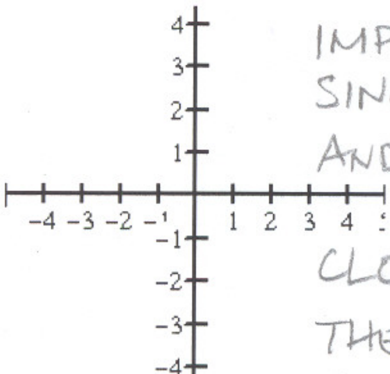
C IS A CRITICAL NUMBER OF f
IF C IS IN THE DOMAIN OF f
AND $f'(c)=0$ OR $f'(c)$ IS UNDEFINED

State the definition of "global minimum". **SCORE: ___ / 2 POINTS**

f HAS A GLOBAL MINIMUM ON A SET D AT C
IF $f(c) \leq f(x)$ FOR ALL $x \in D$

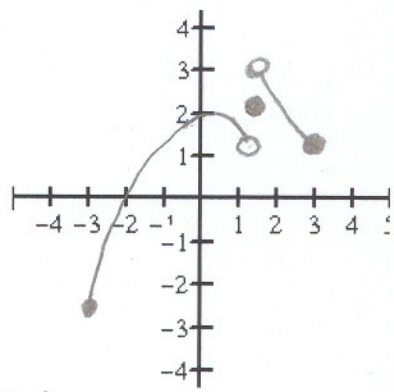
Sketch graphs of functions which satisfy the following properties, or explain **very briefly** why no such function exists. **SCORE: ___ / 4 POINTS**

[a] f is continuous on $[0, 5]$,
f has a global and local maximum at $x = 3$,
f has a local minimum at $x = 2$,
and f has no global minimum on $[0, 5]$.



IMPOSSIBLE.
SINCE f IS CONT.
AND $[0,5]$ IS
CLOSED, BY XVT,
THERE MUST BE
A GLOBAL MINIMUM

[b] g is defined on $[-3, 3]$,
g has a local maximum at $x = 0$ but no global maximum,
and g has a global minimum that is not a local minimum.
g should have no other local or global extrema.



Find the global extrema for $f(x) = x^{\frac{2}{3}}(x-4)$ on $[-1, 3]$. **SCORE: ___ / 5 POINTS**

$f(x) = x^{\frac{5}{3}} - 4x^{\frac{2}{3}}$

$f'(x) = \frac{5}{3}x^{\frac{2}{3}} - \frac{8}{3}x^{-\frac{1}{3}} = \frac{1}{3}x^{-\frac{1}{3}}(5x-8)$ IS UNDEFINED
@ $x = 0 \in$ DOMAIN

$f'(x) = 0$ @ $x = \frac{8}{5} = 1.6 \in$ DOMAIN

$f(-1) = (-1)^{\frac{2}{3}}(-1-4) = -5$
$f(0) = 0^{\frac{2}{3}}(0-4) = 0$
$f(1.6) = 1.6^{\frac{2}{3}}(1.6-4) = -3.36$
$f(3) = 3^{\frac{2}{3}}(3-4) = -2.1$

GLOBAL MAX AT $(0,0)$
MIN AT $(-1,-5)$

State the Mean Value Theorem. **SCORE: ___ / 2 POINTS**

IF f IS CONTINUOUS ON $[a,b]$ AND DIFFERENTIABLE ON (a,b), THEN THERE IS A $c \in (a,b)$ SUCH THAT $f'(c) = \frac{f(b)-f(a)}{b-a}$

Consider $f(x) = x^{\frac{2}{3}}$ on the interval $[-1, 8]$. **SCORE: ___ / 5 POINTS**

[a] Show that there is no value of c which satisfies the conclusion of the Mean Value Theorem.

$$\frac{f(8)-f(-1)}{8--1} = \frac{4-1}{9} = \frac{1}{3}$$

$$f'(c) = \frac{2}{3}x^{-\frac{1}{3}} = \frac{1}{3} \text{ IF } \frac{2}{\sqrt[3]{x}} = 1$$

$$\text{IE. } 2 = \sqrt[3]{x}$$

$$\text{IE. } x = 8 \notin (0,8)$$

[b] Why does this not contradict the Mean Value Theorem ?

$f'(0)$ DOES NOT EXIST

SO f' IS NOT DIFFERENTIABLE ON $(-1,8)$

Let $y = \frac{4}{1+x^2}$, $x = 1$ and $\Delta x = -0.5$. **SCORE: ___ / 5 POINTS**

[a] Find Δy and dy.

$$\Delta y = y(1+{-0.5}) - y(1) = \frac{4}{1+(\frac{1}{2})^2} - \frac{4}{1+1^2} = \frac{16}{5} - 2 = \frac{6}{5}$$

$$dy = \frac{-8x}{(1+x^2)^2}dx = \frac{-8(1)}{(1+1^2)^2}(-0.5) = \frac{4}{4} = 1$$

[b] Using the differential in [a], complete the following sentence:

When x equals 0.5, y is approximately equal to 3.

$x + dx = 0.5$ $y + dy = 2 + 1$

Use a linear approximation to estimate $15.92^{\frac{3}{4}}$. **SCORE: ___ / 5 POINTS**

$f(x) = x^{\frac{3}{4}}$ NEAR $x = 16$

$$f'(x) = \frac{3}{4}x^{-\frac{1}{4}}$$

$$L(x) = f(16) + f'(16)(x-16)$$

$$= 16^{\frac{3}{4}} + \frac{3}{4}16^{-\frac{1}{4}}(x-16)$$

$$= 8 + \frac{3}{8}(x-16)$$

$$15.92^{\frac{3}{4}} = f(15.92)$$

$$\approx L(15.92)$$

$$= 8 + \frac{3}{8}(15.92-16)$$

$$= 8 + \frac{3}{8}(-0.08)$$

$$= 8 - 0.03 = 7.97$$