Math 1A (7:30am – 8:20am)
Quiz 7
Fri Nov 19, 2010

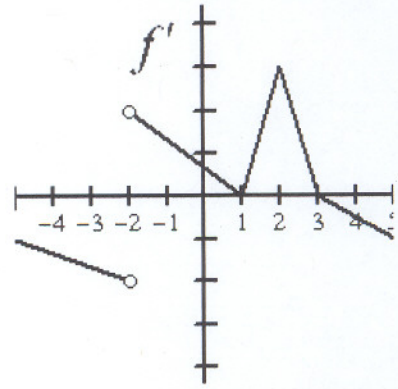
NAME YOU ASKED TO BE CALLED: ____________________

SCORE: ___ / 30 POINTS

NO CALCULATORS ALLOWED
SHOW PROPER WORK / USE PROPER NOTATION / SIMPLIFY YOUR ANSWERS

The graph of the derivative f' of a **continuous** function f is shown on the right.

SCORE: ___ / 10 POINTS



[a] Write "OK" if you understand that the graph shows f' and **NOT** f, but that all the questions below are about f and **NOT** f'.

OK

[b] Find the x – coordinates of all critical numbers of f. Explain your answer(s) **very briefly**.

$f' = 0$ AT $x = 1, 3$

f' UNDEFINED AT $x = -2$

[c] Find all intervals over which f is increasing. Explain your answer(s) **very briefly**.

$f' > 0$ ON $[-2, 3]$

[d] Find the x – coordinates of all local minima of f. Explain your answer(s) **very briefly**.

f' CHANGES FROM NEGATIVE TO POSITIVE AT $x = -2$

[e] Find all intervals over which f is concave up. Explain your answer(s) **very briefly**.

f' INCREASING ON $[1, 2]$

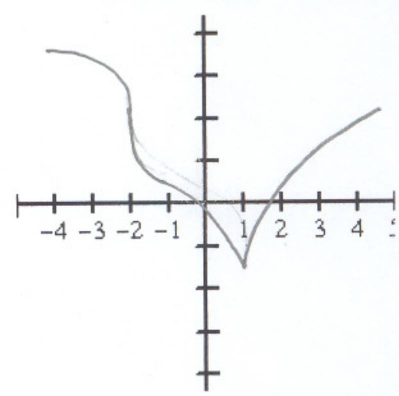
[f] Find the x – coordinates of all inflection points of f. Explain your answer(s) **very briefly**.

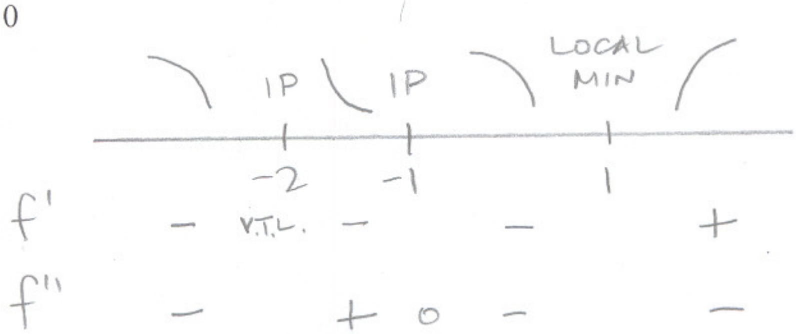
f' CHANGES FROM INCREASING TO DECREASING AT $x = 2$
DECREASING TO INCREASING $x = 1$

Sketch the graph of a **continuous** function that satisfies all the given conditions.

SCORE: ___ / 5 POINTS

$f'(x) < 0$ if $x < -2$ or $-2 < x < 1$,
$f'(x) > 0$ if $x > 1$,
$\lim\limits_{x \to -2} f'(x) = -\infty$,
$f''(x) < 0$ if $x < -2$ or $|x| < 1$ or $x > 1$,
$f''(x) > 0$ if $-2 < x < -1$,
$f''(-1) = 0$





Find the following limits. **SCORE: ___ / 7 POINTS**

The answer should be a number, ∞ or $-\infty$. Write DNE only if the other possibilities do not apply.

[a] $\lim_{x \to 0^+} \frac{1-\cos x}{x-\sin x}$ $\quad \frac{1-1}{0-0} \to \frac{0}{0}$

$= \lim_{x \to 0^+} \frac{\sin x}{1-\cos x} \quad \frac{0}{1-1} \to \frac{0}{0}$

$= \lim_{x \to 0^+} \frac{\cos x}{\sin x} \quad \frac{1}{0^+}$

$= \infty$

[b] $\lim_{x \to 0} \frac{1-x+e^{-x}}{x^2}$ $\quad \frac{1-0+1}{0^+} \to \frac{2}{0^+}$

$= \infty$

Let $f(x) = x^4 e^{-x}$. **SCORE: ___ / 8 POINTS**

[a] Find all critical numbers of f.

$f'(x) = 4x^3e^{-x} - x^4e^{-x}$

$= x^3e^{-x}(4-x) = 0$ IF $x = 0, 4$

IS NEVER UNDEFINED

[b] For each critical number, determine what the Second Derivative Test tells you about that critical number.

$f''(x) = 12x^2e^{-x} - 4x^3e^{-x} - 4x^3e^{-x} + x^4e^{-x}$

$= x^2e^{-x}(12 - 8x + x^2)$

$f''(0) = 0$ NO CONCLUSION

$f''(4) < 0$ LOCAL MAX

[c] Find the inflection points of f.

$f''(x) = x^2e^{-x}(6-x)(2-x) = 0$ IF $x = 0, 2, 6$

IS NEVER UNDEFINED

		0		2		6	
f''	+		+		−		+
x^2e^{-x}	+		+		+		+
$6-x$	+		+		+		−
$2-x$	+		+		−		−

INFLECTION POINTS AT $x = 2, 6$