Math 1A (7:30am – 8:20am)
Quiz 8
Wed Nov 24, 2010

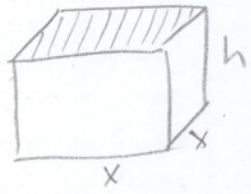
NAME YOU ASKED TO BE CALLED: _____

SCORE: ___ / 30 POINTS

NO CALCULATORS ALLOWED
SHOW PROPER WORK / USE PROPER NOTATION / SIMPLIFY YOUR ANSWERS

If 900 square inches of metal is available to make a box with a square base and an open top, find the volume of the largest possible box.

SCORE: ___ / 12 POINTS



MAXIMIZE V = VOLUME BY CHANGING x, h

$V = x^2 h$

$x^2 + 4xh = 900$

$h = \frac{900 - x^2}{4x}$

$V = x^2\left(\frac{900 - x^2}{4x}\right) = \frac{1}{4}(900x - x^3)$

$0 \le x \le 30$

$V' = \frac{1}{4}(900 - 3x^2)$ IS NEVER UNDEFINED

$= 0$ IF $x = \sqrt{300} = 10\sqrt{3}$

$V(0) = 0$

$V(30) = 0$

$V(10\sqrt{3}) = \frac{1}{4}(9000\sqrt{3} - 3000\sqrt{3})$

$= 1500\sqrt{3}$ in^2 IS THE MAXIMUM VOLUME

OR BY FIRST DERIVATIVE TEST

V' : + on $(0, 10\sqrt{3})$, − on $(10\sqrt{3}, 30)$

$x = 10\sqrt{3}$ IS THE GLOBAL MAX

Find the following limits. **SCORE: ___ / 18 POINTS**

The answer should be a number, ∞ or $-\infty$. Write DNE only if the other possibilities do not apply.

[a] $\lim_{x\to 0^+}(1+4x)^{\cot x}$ 1^∞

FIRST FIND $\lim_{x\to 0^+} \ln(1+4x)^{\cot x}$

$= \lim_{x\to 0^+} \cot x \ln(1+4x)$ $\infty \cdot 0$

$= \lim_{x\to 0^+} \dfrac{\ln(1+4x)}{\tan x}$

$= \lim_{x\to 0^+} \dfrac{\frac{1}{1+4x}\cdot 4}{\sec^2 x}$

$= \dfrac{1\cdot 4}{1^2}$

$= 4$

so $\lim_{x\to 0^+}(1+4x)^{\cot x} = e^4$

[b] $\lim_{x\to 0^+} \dfrac{\ln x}{\cos x - 1}$ $\dfrac{-\infty}{0^-}$

$= \infty$

[c] $\lim_{x\to\infty} \dfrac{\ln x}{\sqrt{x}}$ $\dfrac{\infty}{\infty}$

$= \lim_{x\to\infty} \dfrac{\frac{1}{x}}{\frac{1}{2\sqrt{x}}}$

$= \lim_{x\to\infty} \dfrac{2}{\sqrt{x}}$ $\dfrac{2}{\infty}$

$= 0$

[d] $\lim_{x\to\infty} x \sin\dfrac{2}{x}$ $\infty \cdot 0$

$= \lim_{x\to\infty} \dfrac{\sin\frac{2}{x}}{\frac{1}{x}}$

$= \lim_{x\to\infty} \dfrac{\cos\frac{2}{x}\cdot\frac{-2}{x^2}}{-\frac{1}{x^2}}$

$= \lim_{x\to\infty} 2\cos\dfrac{2}{x}$

$= 2\cdot 1$

$= 2$