Math 114
Midterm 1
Wed Apr 28, 2010

Name: _____

SCORE: _____ / 140 POINTS

- **ALL PROBLEMS MUST BE SOLVED ALGEBRAICALLY TO EARN CREDIT (NO GUESS & CHECK)**
- **PUT A BOX AROUND EACH FINAL ANSWER**
- **SHOW COMPLETE AND PROPER WORK TO EARN FULL CREDIT**

Fill in the blanks: The equation of the vertical asymptote for $y = \dfrac{4-9x}{15x+10}$ is $x = -\frac{2}{3}$.

SCORE: ___ / 10 POINTS

$15x+10=0$

$x = \frac{-10}{15} = -\frac{2}{3}$

The equation of the horizontal asymptote for $y = \dfrac{4-9x}{15x+10}$ is $y = -\frac{3}{5}$.

$y \approx \frac{-9x}{15x} = -\frac{3}{5}$

Simplify: $\dfrac{\frac{3}{x+3}-\frac{2}{x}}{\frac{4}{x}+\frac{5}{x+3}} \cdot \dfrac{x(x+3)}{x(x+3)}$

SCORE: ___ / 10 POINTS

$= \dfrac{3x-2(x+3)}{4(x+3)+5x} = \dfrac{3x-2x-6}{4x+12+5x} = \dfrac{x-6}{9x+12} = \boxed{\dfrac{x-6}{3(3x+4)}}$

Subtract and simplify: $\dfrac{3x^2-3x+5}{x^2-4x-12} - \dfrac{2x^2+5x-7}{x^2-4x-12}$

SCORE: ___ / 10 POINTS

$= \dfrac{x^2-8x+12}{x^2-4x-12} = \dfrac{(x-2)(x-6)}{(x+2)(x-6)} = \boxed{\dfrac{x-2}{x+2}}$

Simplify: $\dfrac{\frac{3}{x-5}-1}{2+\frac{4}{x-5}} \cdot \dfrac{x-5}{x-5}$

SCORE: ___ / 10 POINTS

$= \dfrac{3-(x-5)}{2(x-5)+4} = \dfrac{3-x+5}{2x-10+4} = \dfrac{8-x}{2x-6} = \boxed{\dfrac{8-x}{2(x-3)}}$

Multiply and simplify: $\dfrac{x^2+5x+6}{x^2-3x-10}\cdot\dfrac{x^2-2x-15}{2x^2-18}$

SCORE: ___ / 10 POINTS

$$= \frac{(x+2)(x+3)}{(x+2)(x-5)} \cdot \frac{(x+3)(x-5)}{2(x^2-9)}$$

$$= \frac{(x+2)(x+3)}{(x+2)(x-5)} \cdot \frac{(x+3)(x-5)}{2(x+3)(x-3)} = \boxed{\frac{x+3}{2(x-3)}}$$

Subtract and simplify: $\dfrac{x-2}{x^2-11x+28}-\dfrac{x+3}{x^2-8x+7}$

SCORE: ___ / 12 POINTS

$$= \frac{x-2}{(x-4)(x-7)} - \frac{x+3}{(x-1)(x-7)}$$

$$= \frac{(x-2)(x-1)-(x+3)(x-4)}{(x-4)(x-7)(x-1)}$$

$$= \frac{x^2-3x+2-(x^2-x-12)}{(x+4)(x-7)(x-1)}$$

$$= \frac{-2x+14}{(x+4)(x-7)(x-1)}$$

$$= \frac{-2(x-7)}{(x+4)(x-7)(x-1)}$$

$$= \boxed{\frac{-2}{(x+4)(x-1)}}$$

Write a proportion for, but **DO NOT SOLVE**:

SCORE: ___ / 4 POINTS

If 253 US dollars is equivalent to 274 Swiss francs, how many Swiss francs is 187 US dollars equivalent to ?

$$\boxed{\frac{253}{274} = \frac{187}{x}}$$

Add and simplify: $\dfrac{2}{x+5}+\dfrac{x-1}{x+7}$

SCORE: ___ / 10 POINTS

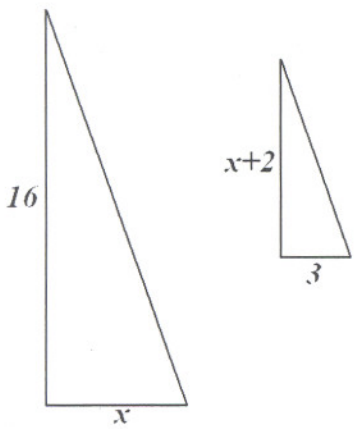
$$= \frac{2(x+7)+(x-1)(x+5)}{(x+5)(x+7)}$$

$$= \frac{2x+14+x^2+4x-5}{(x+5)(x+7)}$$

$$= \frac{x^2+6x+9}{(x+5)(x+7)} = \boxed{\frac{(x+3)^2}{(x+5)(x+7)}}$$

Solve for x in the following similar triangles:

SCORE: ___ / 10 POINTS



$$\frac{16}{x} = \frac{x+2}{3}$$

$$48 = x^2 + 2x$$

$$0 = x^2 + 2x - 48$$

$$0 = (x+8)(x-6)$$

$$x = -8, \boxed{6}$$

Solve: The height of a cake varies directly as the amount of batter available and inversely as the base area of its baking pan. Baking 3 cups of batter in a 48 square inch pan results in a 2 inch tall cake. How tall is a cake created by baking 4 cups of batter in a 32 square inch pan ?

SCORE: ___ / 12 POINTS

FOR FULL CREDIT, YOU MUST IDENTIFY WHAT ALL YOUR VARIABLES REPRESENT, FIND THE EQUATION CONNECTING THEM, AND SUMMARIZE YOUR FINAL ANSWER IN A SENTENCE.

h = HEIGHT OF CAKE (INCH)
b = AMOUNT OF BATTER (CUP)
a = AREA OF BASE (SQUARE INCH)

$h = \frac{kb}{a}$

$2 = \frac{k(3)}{48}$

$3k = 96$

$k = 32$

$h = \frac{32b}{a}$

$h = \frac{32(4)}{32}$

$h = 4$

4 CUPS OF BATTER BAKED IN A 32 SQUARE INCH PAN GIVES A 4 INCH TALL CAKE

Solve for x: $12x^2\left(\frac{1}{12} + \frac{2}{3x^2}\right) = \left(\frac{1}{2x}\right)12x^2$ **CHECK YOUR ANSWER(S)**

SCORE: ___ / 12 POINTS

$x^2 + 8 = 6x$

$x^2 - 6x + 8 = 0$

$(x-2)(x-4) = 0$

$\boxed{x = 2, 4}$

CHECK:

$x = 2$

$\frac{1}{12} + \frac{2}{3(2)^2} = \frac{1}{12} + \frac{2}{12} = \frac{3}{12} = \frac{1}{4}$

$\frac{1}{2(2)} = \frac{1}{4}$ ✓

$x = 4$

$\frac{1}{12} + \frac{2}{3(4)^2} = \frac{1}{12} + \frac{2}{48} = \frac{1}{12} + \frac{1}{24} = \frac{2}{24} + \frac{1}{24}$

$= \frac{3}{24} = \frac{1}{8}$

$\frac{1}{2(4)} = \frac{1}{8}$ ✓

Divide and simplify: $\dfrac{6x^2+12x}{9x^2-4} \div \dfrac{4x+8}{9x-6}$ **SCORE: ___ / 10 POINTS**

$= \dfrac{6x^2+12x}{9x^2-4} \cdot \dfrac{9x-6}{4x+8}$

$= \dfrac{3}{} \dfrac{6x(x+2)}{(3x+2)(3x-2)} \cdot \dfrac{3(3x-2)}{4(x+2)}$ $_2$

$= \boxed{\dfrac{9x}{2(3x+2)}}$

Simplify: $\dfrac{3x^2-5x-12}{2x^2-x-15}$ **SCORE: ___ / 10 POINTS**

$= \dfrac{(3x+4)(x-3)}{(2x+5)(x-3)}$

$= \boxed{\dfrac{3x+4}{2x+5}}$

Solve: A number divided by twelve is equal to six divided by one less than that number. Find the number. **CHECK YOUR ANSWER(S).** **SCORE: ___ / 10 POINTS**

$\dfrac{x}{12} = \dfrac{6}{x-1}$

$x^2 - x = 72$

$x^2 - x - 72 = 0$

$(x-9)(x+8) = 0$

$\boxed{x = 9, -8}$

CHECK:

$x = 9$

$\dfrac{9}{12} = \dfrac{3}{4}$

$\dfrac{6}{9-1} = \dfrac{6}{8} = \dfrac{3}{4}$ ✓

$x = -8$

$\dfrac{-8}{12} = -\dfrac{2}{3}$

$\dfrac{6}{-8-1} = \dfrac{6}{-9} = -\dfrac{2}{3}$ ✓