Math 1B (7:30am – 8:20am)
Midterm 1 Version A
Tue Apr 27, 2010

SCORE: ___ / 150 POINTS

What month is your birthday ? ___ ___
What are the first 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___
What are the last 2 digits of your social security number ? ___ ___
[IF YOU DO NOT HAVE A SOCIAL SECURITY NUMBER, USE YOUR STUDENT ID NUMBER]

NO CALCULATORS ALLOWED
YOU MUST SHOW PROPER CALCULUS LEVEL WORK

State the definition of "definite integral". **[Same question and answer as quiz #1 and #2]** **SCORE: ___ / 5 POINTS**

SEE PREVIOUS QUIZ KEYS

State the Fundamental Theorem of Calculus Parts 1 and 2. **SCORE: ___ / 10 POINTS**

SEE PREVIOUS QUIZ KEYS

If $\operatorname{csch} x = 2$, find $\tanh x$. **SCORE: ___ / 15 POINTS**

$\frac{1}{\sinh x} = 2 \Rightarrow \sinh x = \frac{1}{2}$

$\cosh^2 x - 1 = \sinh^2 x = \frac{1}{4}$

$\cosh^2 x = \frac{5}{4}$

$\cosh x = \frac{\sqrt{5}}{2}$ SINCE $\cosh x > 0$

$\tanh x = \frac{\sinh x}{\cosh x} = \frac{\frac{1}{2}}{\frac{\sqrt{5}}{2}} = \frac{1}{\sqrt{5}}$

If $F(x) = \int_1^x f(t)\,dt$ and $f(x) = \int_{\cosh^{-1} x}^{3} e^{\sqrt{t}}\,dt$, find $F''(x)$. **SCORE: ___ / 15 POINTS**

$F'(x) = f(x)$

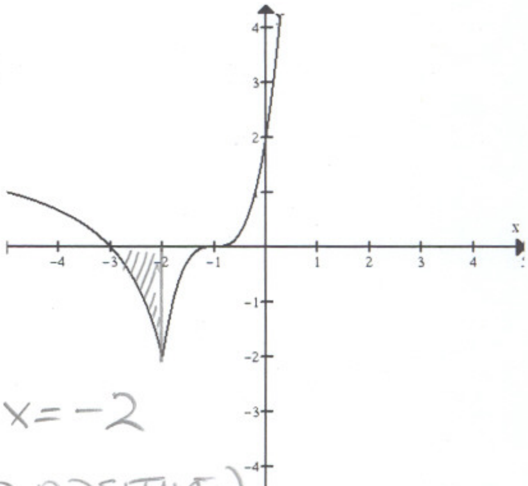
$$F''(x) = f'(x) = \frac{d}{dx}\int_{\cosh^{-1}x}^{3} e^{\sqrt{t}}\,dt$$

$$= -\frac{d}{dx}\int_3^{\cosh^{-1}x} e^{\sqrt{t}}\,dt$$

$$= -\frac{d}{d\cosh^{-1}x}\int_3^{\cosh^{-1}x} e^{\sqrt{t}}\,dt \cdot \frac{d\cosh^{-1}x}{dx}$$

$$= -e^{\sqrt{\cosh^{-1}x}} \cdot \frac{1}{\sqrt{x^2-1}} = -\frac{e^{\sqrt{\cosh^{-1}x}}}{\sqrt{x^2-1}}$$

The graph of $f(t)$ is shown below. Let $g(x) = \int_{-2}^{x} f(t)\,dt$. **SCORE: ___ / 15 POINTS**



[a] Is $g(-3)$ positive or negative ? Explain your reasoning.

$g(-3) = \int_{-2}^{-3} f(t)\,dt = -\int_{-3}^{-2} f(t)\,dt > 0$

SINCE $\int_{-3}^{-2} f(t)\,dt$ IS THE NEGATIVE OF THE SHADED AREA

[b] At what value(s) of x does g have an inflection point ? Explain your reasoning.

$g' = f$ CHANGES FROM DECREASING TO INCREASING @ $x = -2$

(OR $g'' = f'$ CHANGES FROM NEGATIVE TO POSITIVE)

Use the definition of the definite integral, with right endpoints, to evaluate $\int_{-2}^{1} (x^2 + 4x)\,dx$. **SCORE: ___ / 20 POINTS**

DO NOT USE THE FUNDAMENTAL THEOREM OF CALCULUS.

$\lim\limits_{n\to\infty} \sum_{i=1}^{n} \left(\left(-2 + \frac{3i}{n}\right)^2 + 4\left(-2 + \frac{3i}{n}\right)\right)\frac{3}{n}$

$= \lim\limits_{n\to\infty} \sum_{i=1}^{n} \left(-2 + \frac{3i}{n}\right)\left(-2 + \frac{3i}{n} + 4\right)\frac{3}{n}$

$= \lim\limits_{n\to\infty} \frac{3}{n} \sum_{i=1}^{n} \left(-2 + \frac{3i}{n}\right)\left(2 + \frac{3i}{n}\right)$

$= \lim\limits_{n\to\infty} \frac{3}{n} \sum_{i=1}^{n} \left(\frac{9i^2}{n^2} - 4\right)$

$= \lim\limits_{n\to\infty} \frac{3}{n} \left[\frac{9}{n^2} \sum_{i=1}^{n} i^2 - \sum_{i=1}^{n} 4\right]$

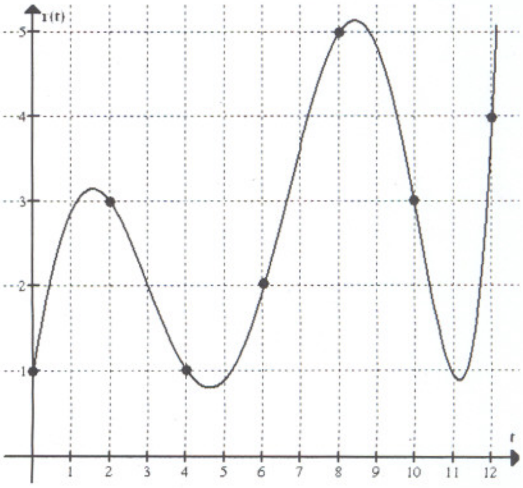
$= \lim\limits_{n\to\infty} \frac{3}{\cancel{n}} \left[\frac{\cancel{9}^{3}}{n^2} \frac{\cancel{n}(n+1)(2n+1)}{\cancel{6}_{2}} - 4\cancel{n}\right]$

$= \lim\limits_{n\to\infty} 3\left[\frac{3(n+1)(2n+1)}{2n^2} - 4\right]$

$= 3\left(\frac{6}{2} - 4\right)$

$= -3$

The graph shows the rate $r(t)$ at which water is poured into a bathtub (in gallons/minute) at various times (in minutes). At time $t = 2$ minutes, the bathtub contained 6 gallons of water. Estimate the amount of water in the bathtub at time $t = 10$ minutes using 4 subintervals and left endpoints. **SCORE: ___ / 15 POINTS**



IF $v(t)$ = VOLUME OF WATER IN TUB AT TIME t

THEN $v'(t) = r(t)$

SO $v(10) - v(2) = \int_2^{10} r(t)\,dt$

$v(10) = v(2) + \int_2^{10} r(t)\,dt$

$v(10) \approx 6 + [3 + 1 + 2 + 5]\,2$

$= 28$ gallons

Evaluate the following integrals. Show all relevant work. **SCORE: ___ / 55 POINTS**

[a] $\displaystyle\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{t+t^3}{\cos t}\,dt$

$$\frac{(-t)+(-t)^3}{\cos(-t)} = \frac{-t-t^3}{\cos t} = -\left(\frac{t+t^3}{\cos t}\right)$$

SINCE THE INTEGRAND IS ODD ON $[-\frac{\pi}{4}, \frac{\pi}{4}]$

THE INTEGRAL $= 0$

[b] $\displaystyle\int_1^8 \frac{(3+\sqrt[3]{t})^2}{t}\,dt$

$$= \int_1^8 \frac{9+6t^{\frac{1}{3}}+t^{\frac{2}{3}}}{t}\,dt$$

$$= \int_1^8 \left(\frac{9}{t} + 6t^{-\frac{2}{3}} + t^{-\frac{1}{3}}\right)dt$$

$$= \left(9\ln|t| + 18t^{\frac{1}{3}} + \frac{3}{2}t^{\frac{2}{3}}\right)\Big|_1^8$$

$$= \left(9\ln 8 + 18(2) + \frac{3}{2}(4)\right) - \left(9\ln 1 + 18 + \frac{3}{2}\right)$$

$$= 9\ln 8 + 18 + \frac{9}{2}$$

$$= 9\ln 8 + \frac{45}{2}$$

[c] $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\sin 2t}{2-\cos^2 t}\,dt$

$u = 2-\cos^2 t$; $t = \frac{\pi}{2} \Rightarrow u = 2$; $t = 0 \Rightarrow u = 1$

$$\frac{du}{dt} = (-2\cos t)(-\sin t)$$

$$du = \sin 2t\,dt$$

$$\int_1^2 \frac{1}{u}\,du$$

$$= \ln|u|\,\Big|_1^2$$

$$= \ln 2 - \ln 1$$

$$= \ln 2$$

[d] $\displaystyle\int \frac{\sin(\tanh^{-1} t)}{(t^2-1)}\,dt$

$$u = \tanh^{-1} t$$

$$\frac{du}{dt} = \frac{1}{1-t^2}$$

$$-du = \frac{1}{t^2-1}\,dt$$

$$\int -\sin u\,du$$

$$= \cos u + C$$

$$= \cos(\tanh^{-1} t) + C$$