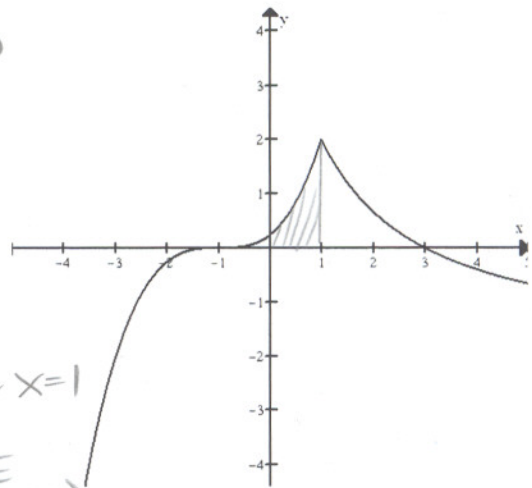
Math 1B (7:30am – 8:20am)
Midterm 1 Version B
Tue Apr 27, 2010

SCORE: ___ / 150 POINTS

What month is your birthday ? ___ ___
What are the first 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___
What are the last 2 digits of your social security number ? ___ ___
[IF YOU DO NOT HAVE A SOCIAL SECURITY NUMBER, USE YOUR STUDENT ID NUMBER]

NO CALCULATORS ALLOWED
YOU MUST SHOW PROPER CALCULUS LEVEL WORK

The graph of $f(t)$ is shown below. Let $g(x) = \int_1^x f(t)\, dt$. **SCORE: ___ / 15 POINTS**



[a] Is $g(0)$ positive or negative ? Explain your reasoning.

$g(0) = \int_1^0 f(t)\,dt = -\int_0^1 f(t)\,dt < 0$

SINCE $\int_0^1 f(t)\,dt$ IS SHADED AREA

[b] At what value(s) of x does g have an inflection point ? Explain your reasoning.

$g' = f$ CHANGES FROM INCREASING TO DECREASING @ $x=1$

(OR $g'' = f'$ CHANGES FROM POSITIVE TO NEGATIVE)

Use the definition of the definite integral, with right endpoints, to evaluate $\int_{-3}^{1} (x^2 + 6x)\, dx$. **SCORE: ___ / 20 POINTS**

DO NOT USE THE FUNDAMENTAL THEOREM OF CALCULUS.

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\left(-3+\frac{4i}{n}\right)^2 + 6\left(-3+\frac{4i}{n}\right)^2\right)\frac{4}{n}$$

$$= \lim_{n\to\infty} \frac{4}{n} \sum_{i=1}^{n} \left(-3+\frac{4i}{n}\right)\left(-3+\frac{4i}{n}+6\right)$$

$$= \lim_{n\to\infty} \frac{4}{n} \sum_{i=1}^{n} \left(-3+\frac{4i}{n}\right)\left(3+\frac{4i}{n}\right)$$

$$= \lim_{n\to\infty} \frac{4}{n} \sum_{i=1}^{n} \left(\frac{16i^2}{n^2} - 9\right)$$

$$= \lim_{n\to\infty} \frac{4}{n} \left[\frac{16}{n^2}\sum_{i=1}^{n} i^2 - \sum_{i=1}^{n} 9\right]$$

$$= \lim_{n\to\infty} \frac{4}{\cancel{n}} \left[\frac{\cancel{16}^{\,8}}{n^2} \frac{\cancel{n}(n+1)(2n+1)}{\cancel{6}_{\,3}} - 9\cancel{n}\right]$$

$$= \lim_{n\to\infty} 4\left[\frac{8(n+1)(2n+1)}{3n^2} - 9\right]$$

$$= 4\left[\frac{16}{3} - 9\right]$$

$$= -\frac{44}{3}$$

State the definition of "definite integral". **[Same question and answer as quiz #1 and #2]** **SCORE: ___ / 5 POINTS**

SEE PREVIOUS QUIZ KEYS

If $\operatorname{csch} x = 3$, find $\tanh x$. **SCORE: ___ / 15 POINTS**

$\coth^2 x - 1 = \operatorname{csch}^2 x = 9$

$\coth^2 x = 10$

$\coth x = \pm\sqrt{10}$

SINCE $\operatorname{csch} x > 0$, THEREFORE $\sinh x = \frac{1}{\operatorname{csch} x} > 0$

AND SINCE $\cosh x > 0$ FOR ALL x

$\coth x = \frac{\sinh x}{\cosh x} > 0$ IE. $\coth x = \sqrt{10}$

$\tanh x = \frac{1}{\coth x} = \frac{1}{\sqrt{10}}$

If $F(x) = \int_1^x f(t)\,dt$ and $f(x) = \int_{\sinh^{-1} x}^{3} e^{\sqrt{t}}\,dt$, find $F''(x)$. **SCORE: ___ / 15 POINTS**

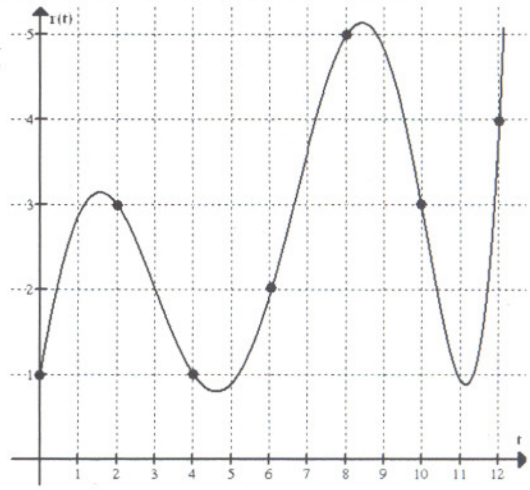
$F'(x) = f(x)$

$F''(x) = f'(x) = \frac{d}{dx}\int_{\sinh^{-1}x}^{3} e^{\sqrt{t}}\,dt = -\frac{d}{dx}\int_{3}^{\sinh^{-1}x} e^{\sqrt{t}}\,dt$

$= -\frac{d}{d\sinh^{-1}x}\int_{3}^{\sinh^{-1}x} e^{\sqrt{t}}\,dt \cdot \frac{d\sinh^{-1}x}{dx}$

$= -e^{\sqrt{\sinh^{-1}x}} \cdot \frac{1}{\sqrt{1+x^2}} = -\frac{e^{\sqrt{\sinh^{-1}x}}}{\sqrt{1+x^2}}$

The graph shows the rate $r(t)$ at which water is poured into a bathtub (in gallons/minute) at various times (in minutes). At time $t = 4$ minutes, the bathtub contained 3 gallons of water. Estimate the amount of water in the bathtub at time $t = 12$ minutes using 4 subintervals and right endpoints. **SCORE: ___ / 15 POINTS**



IF $v(t)$ = VOLUME OF WATER IN TUB AT TIME t

THEN $v'(t) = r(t)$

SO $v(12) - v(4) = \int_4^{12} r(t)\,dt$

$v(12) = v(4) + \int_4^{12} r(t)\,dt$

$v(12) \approx 3 + [2+5+3+4]2$

$= 31$ gallons

State the Fundamental Theorem of Calculus Part 1 and the Net Change Theorem. **SCORE: ___ / 10 POINTS**

SEE PREVIOUS QUIZ KEYS

Evaluate the following integrals. Show all relevant work. **SCORE: ___ / 15 POINTS**

[a] $\int \frac{\sin(\tanh^{-1} t)}{(t^2 - 1)}\, dt$

SEE 7:30 VERSION A KEY

[b] $\int_0^{\frac{\pi}{2}} \frac{\sin 2t}{2 + \cos^2 t}\, dt$

$u = 2 + \cos^2 t$; $t = \frac{\pi}{2} \Rightarrow u = 2$; $t = 0 \Rightarrow u = 3$

$\frac{du}{dt} = (2\cos t)(-\sin t)$

$-du = \sin 2t\, dt$

$-\int_3^2 \frac{1}{u}\, du$

$= \int_2^3 \frac{1}{u}\, du$

$= \ln|u| \Big|_2^3$

$= \ln 3 - \ln 2$

$= \ln \frac{3}{2}$

[c] $\int_1^8 \frac{(3 - \sqrt[3]{t})^2}{t}\, dt$

$= \int_1^8 \frac{9 - 6t^{\frac{1}{3}} + t^{\frac{2}{3}}}{t}\, dt$

$= \int_1^8 \left(\frac{9}{t} - 6t^{-\frac{2}{3}} + t^{-\frac{1}{3}}\right) dt$

$= \left(9\ln|t| - 18t^{\frac{1}{3}} + \frac{3}{2}t^{\frac{2}{3}}\right)\Big|_1^8$

$= \left(9\ln 8 - 18(2) + \frac{3}{2}(4)\right) - \left(9\ln 1 - 18 + \frac{3}{2}\right)$

$= 9\ln 8 - 18 + \frac{9}{2}$

$= 9\ln 8 - \frac{27}{2}$

[d] $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan t}{1 + t^4}\, dt$

$\frac{\tan(-t)}{1 + (-t)^4} = -\frac{\tan t}{1 + t^4}$

SINCE THE INTEGRAND IS ODD ON $[-\frac{\pi}{4}, \frac{\pi}{4}]$

THE INTEGRAL IS 0