Math 1B (9:30am – 10:20am)
Midterm 1 Version C
Tue Apr 27, 2010

SCORE: ___ / 150 POINTS

What month is your birthday ? ___ ___
What are the first 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___
What are the last 2 digits of your social security number ? ___ ___
[IF YOU DO NOT HAVE A SOCIAL SECURITY NUMBER, USE YOUR STUDENT ID NUMBER]

NO CALCULATORS ALLOWED
YOU MUST SHOW PROPER CALCULUS LEVEL WORK

State the definition of "definite integral". **[Same question and answer as quiz #1 and #2]** **SCORE: ___ / 5 POINTS**

SEE PREVIOUS QUIZ KEYS

State the Fundamental Theorem of Calculus Part 1 and the Net Change Theorem. **SCORE: ___ / 10 POINTS**

SEE PREVIOUS QUIZ KEYS

If $\tanh x = \frac{2}{3}$, find $\operatorname{csch} x$. **SCORE: ___ / 15 POINTS**

$\coth x = \frac{1}{\tanh x} = \frac{3}{2}$

$\operatorname{csch}^2 x = \coth^2 x - 1 = \frac{5}{4}$

$\operatorname{csch} x = \pm \frac{\sqrt{5}}{2}$

SINCE $\tanh x = \frac{\sinh x}{\cosh x}$ AND $\tanh x, \cosh x > 0$

THEREFORE, $\sinh x > 0$ AND $\operatorname{csch} x = \frac{1}{\sinh x} > 0$, SO $\operatorname{csch} x = \frac{\sqrt{5}}{2}$

If $F(x) = \int_{-\cosh^{-1} x}^{\tanh^{-1} x} \csc t \, dt$, find $F'(x)$. **SCORE: ___ / 15 POINTS**

$F(x) = \int_{-\cosh^{-1}x}^{\frac{\pi}{2}} \csc t \, dt + \int_{\frac{\pi}{2}}^{\tanh^{-1}x} \csc t \, dt = -\int_{\frac{\pi}{2}}^{-\cosh^{-1}x} \csc t \, dt + \int_{\frac{\pi}{2}}^{\tanh^{-1}x} \csc t \, dt$

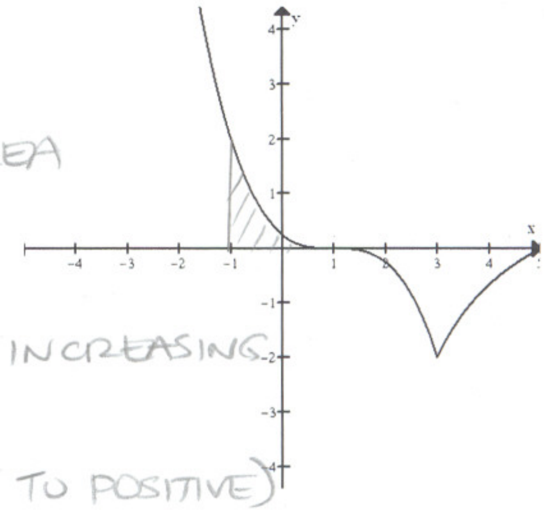
$F'(x) = \frac{d}{dx}\left(-\int_{\frac{\pi}{2}}^{-\cosh^{-1}x} \csc t \, dt\right) + \frac{d}{dx}\int_{\frac{\pi}{2}}^{\tanh^{-1}x} \csc t \, dt$

$= -\frac{d}{d\cosh^{-1}x}\int_{\frac{\pi}{2}}^{-\cosh^{-1}x} \csc t \, dt \cdot \frac{d(\cosh^{-1}x)}{dx} + \frac{d}{d\tanh^{-1}x}\int_{\frac{\pi}{2}}^{\tanh^{-1}x} \csc t \, dt \cdot \frac{d\tanh^{-1}x}{dx}$

$= -\csc(\cosh^{-1}x) \cdot \frac{-1}{\sqrt{x^2-1}} + \csc(\tanh^{-1}x)\frac{1}{1-x^2} = \frac{\csc(\tanh^{-1}x)}{1-x^2} - \frac{\csc(\cosh^{-1}x)}{\sqrt{x^2-1}}$

The graph of $f(t)$ is shown below. Let $g(x) = \int_1^x f(t)\,dt$. **SCORE: ___ / 15 POINTS**

[a] Is $g(-1)$ positive or negative ? Explain your reasoning.



$g(-1) = \int_1^{-1} f(t)\,dt = -\int_{-1}^{1} f(t)\,dt < 0$

SINCE $\int_{-1}^{1} f(t)\,dt$ IS THE SHADED AREA

[b] At what value(s) of x does g have an inflection point ?
Explain your reasoning.

$g' = f$ CHANGES FROM DECREASING TO INCREASING
AT $x = 3$

(OR $g'' = f'$ CHANGES FROM NEGATIVE TO POSITIVE)

Use the definition of the definite integral, with right endpoints, to evaluate $\int_{-2}^{3} (x^2 + 4x)\,dx$. **SCORE: ___ / 20 POINTS**

<u>DO NOT USE THE FUNDAMENTAL THEOREM OF CALCULUS.</u>

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\left(-2+\frac{5i}{n}\right)^2 + 4\left(-2+\frac{5i}{n}\right)\right)\frac{5}{n}$$

$$= \lim_{n\to\infty} \frac{5}{n} \sum_{i=1}^{n} \left(-2+\frac{5i}{n}\right)\left(-2+\frac{5i}{n}+4\right)$$

$$= \lim_{n\to\infty} \frac{5}{n} \sum_{i=1}^{n} \left(-2+\frac{5i}{n}\right)\left(2+\frac{5i}{n}\right)$$

$$= \lim_{n\to\infty} \frac{5}{n} \sum_{i=1}^{n} \left(\frac{25i^2}{n^2} - 4\right)$$

$$= \lim_{n\to\infty} \frac{5}{n} \left[\frac{25}{n^2}\sum_{i=1}^{n} i^2 - \sum_{i=1}^{n} 4\right]$$

$$= \lim_{n\to\infty} \frac{5}{\cancel{n}} \left[\frac{25}{n^2}\frac{\cancel{n}(n+1)(2n+1)}{6} - 4\cancel{n}\right]$$

$$= \lim_{n\to\infty} 5\left[\frac{25(n+1)(2n+1)}{6n^2} - 4\right]$$

$$= 5\left(\frac{50}{6} - 4\right)$$

$$= \frac{65}{3}$$

If f is continuous and $\int_{-1}^{7} f(t)\,dt = 12$, find $\int_0^2 f(7-4t)\,dt$. **SCORE: ___ / 15 POINTS**

$u = 7 - 4t$ $\begin{cases} t = 2 \Rightarrow u = -1 \\ t = 0 \Rightarrow u = 7 \end{cases}$

$\frac{du}{dt} = -4$

$-\frac{1}{4}du = dt$

$$\int_7^{-1} -\frac{1}{4} f(u)\,du = -\frac{1}{4}\int_7^{-1} f(u)\,du = \frac{1}{4}\int_{-1}^{7} f(u)\,du = \frac{1}{4}(12) = 3$$

Evaluate the following integrals. Show all relevant work. **SCORE: ___ / 55 POINTS**

[a] $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\sin 2t}{2+\sin^2 t}\,dt$

$u = 2 + \sin^2 t$, $t = \frac{\pi}{2} \Rightarrow u = 3$, $t = 0 \Rightarrow u = 2$

$\frac{du}{dt} = (2\sin t)\cos t$

$du = \sin 2t\,dt$

$\int_2^3 \frac{1}{u}\,du$

$= \ln|u|\Big|_2^3$

$= \ln 3 - \ln 2$

$= \ln \frac{3}{2}$

[b] $\displaystyle\int_1^{16} \frac{(2-\sqrt[4]{t})^2}{t}\,dt$

$= \int_1^{16} \frac{4 - 4t^{\frac{1}{4}} + t^{\frac{1}{2}}}{t}\,dt$

$= \int_1^{16} \left(\frac{4}{t} - 4t^{-\frac{3}{4}} + t^{-\frac{1}{2}}\right)dt$

$= \left(4\ln|t| - 16t^{\frac{1}{4}} + 2t^{\frac{1}{2}}\right)\Big|_1^{16}$

$= (4\ln 16 - 16(2) + 2(4))$

$- (4\ln 1 - 16 + 2)$

$= 4\ln 16 - 10$

[c] $\displaystyle\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{\tan t}{2+\cos t}\,dt$

$\frac{\tan(-t)}{2+\cos(-t)} = -\frac{\tan t}{2+\cos t}$

SINCE THE INTEGRAND IS ODD ON $[-\frac{\pi}{3}, \frac{\pi}{3}]$, THE INTEGRAL IS 0

[d] $\displaystyle\int \frac{\sec^2(\sinh^{-1} t)}{\sqrt{1+t^2}}\,dt$

$u = \sinh^{-1} t$

$\frac{du}{dt} = \frac{1}{\sqrt{1+t^2}}$

$du = \frac{1}{\sqrt{1+t^2}}\,dt$

$\int \sec^2 u\,du$

$= \tan u + C$

$= \tan(\sinh^{-1} t) + C$