Math 1B (9:30am – 10:20am)
Midterm 1 Version D
Tue Apr 27, 2010

SCORE: ___ / 150 POINTS

What month is your birthday ? ___ ___
What are the first 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___
What are the last 2 digits of your social security number ? ___ ___
[IF YOU DO NOT HAVE A SOCIAL SECURITY NUMBER, USE YOUR STUDENT ID NUMBER]

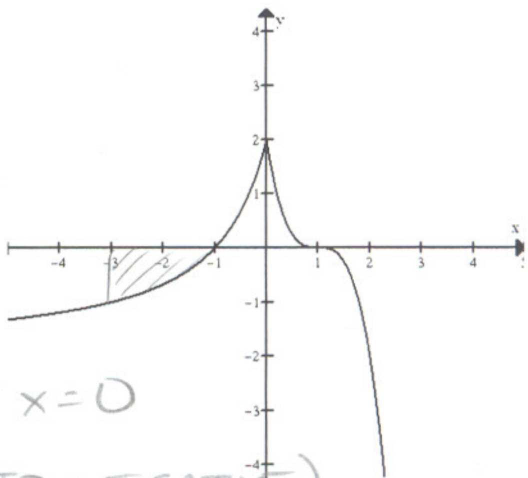
NO CALCULATORS ALLOWED
YOU MUST SHOW PROPER CALCULUS LEVEL WORK

The graph of $f(t)$ is shown below. Let $g(x) = \int_{-1}^{x} f(t)\,dt$. **SCORE: ___ / 15 POINTS**

[a] Is $g(-3)$ positive or negative ? Explain your reasoning.

$g(-3) = \int_{-1}^{-3} f(t)\,dt = -\int_{-3}^{-1} f(t)\,dt > 0$

SINCE $\int_{-3}^{-1} f(t)\,dt$ IS THE NEGATIVE OF THE SHADED AREA



[b] At what value(s) of x does g have an inflection point ? Explain your reasoning.

$g' = f$ CHANGES FROM INCREASING TO DECREASING AT $x = 0$

(OR $g'' = f'$ CHANGES FROM POSITIVE TO NEGATIVE)

Use the definition of the definite integral, with right endpoints, to evaluate $\int_{-3}^{2} (x^2 + 6x)\,dx$. **SCORE: ___ / 20 POINTS**

<u>DO NOT USE THE FUNDAMENTAL THEOREM OF CALCULUS.</u>

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\left(-3+\frac{5i}{n}\right)^2 + 6\left(-3+\frac{5i}{n}\right)\right)\frac{5}{n}$$

$$= \lim_{n\to\infty} \frac{5}{n} \sum_{i=1}^{n} \left(-3+\frac{5i}{n}\right)\left(-3+\frac{5i}{n}+6\right)$$

$$= \lim_{n\to\infty} \frac{5}{n} \sum_{i=1}^{n} \left(-3+\frac{5i}{n}\right)\left(3+\frac{5i}{n}\right)$$

$$= \lim_{n\to\infty} \frac{5}{n} \sum_{i=1}^{n} \left(\frac{25i^2}{n^2} - 9\right)$$

$$= \lim_{n\to\infty} \frac{5}{n} \left[\frac{25}{n^2} \sum_{i=1}^{n} i^2 - \sum_{i=1}^{n} 9\right]$$

$$= \lim_{n\to\infty} \frac{5}{\cancel{n}} \left[\frac{25}{n^2} \frac{\cancel{n}(n+1)(2n+1)}{6} - 9\cancel{n}\right]$$

$$= \lim_{n\to\infty} 5\left[\frac{25(n+1)(2n+1)}{6n^2} - 9\right]$$

$$= 5\left(\frac{50}{6} - 9\right)$$

$$= -\frac{10}{3}$$

State the definition of "definite integral". **<u>[Same question and answer as quiz #1 and #2]</u>** **SCORE: ___ / 5 POINTS**

SEE PREVIOUS QUIZ KEYS

If $\tanh x = \dfrac{3}{4}$, find $\operatorname{csch} x$. **SCORE: ___ / 15 POINTS**

$$\operatorname{sech}^2 x = 1 - \tanh^2 x = \frac{7}{16}$$

$\operatorname{sech} x = \frac{\sqrt{7}}{4}$ (SINCE $\cosh x > 0$ FOR ALL x AND $\operatorname{sech} x = \frac{1}{\cosh x}$)

$$\tanh x = \frac{\sinh x}{\cosh x} \Rightarrow \tanh x = \frac{\operatorname{sech} x}{\operatorname{csch} x} \Rightarrow \operatorname{csch} x = \frac{\operatorname{sech} x}{\tanh x}$$

$$= \frac{\frac{\sqrt{7}}{4}}{\frac{3}{4}} = \frac{\sqrt{7}}{3}$$

If $F(x) = \displaystyle\int_{-\tanh^{-1} x}^{\cosh^{-1} x} \sec t \, dt$, find $F'(x)$. **SCORE: ___ / 15 POINTS**

$$F(x) = \int_{-\tanh^{-1}x}^{0} \sec t \, dt + \int_0^{\cosh^{-1}x} \sec t \, dt = -\int_0^{-\tanh^{-1}x} \sec t \, dt + \int_0^{\cosh^{-1}x} \sec t \, dt$$

$$F'(x) = -\frac{d}{dx}\int_0^{-\tanh^{-1}x} \sec t \, dt + \frac{d}{dx}\int_0^{\cosh^{-1}x} \sec t \, dt$$

$$= -\frac{d}{d\,{-\tanh^{-1}x}}\int_0^{-\tanh^{-1}x} \sec t \, dt \cdot \frac{d(-\tanh^{-1}x)}{dx} + \frac{d}{d\cosh^{-1}x}\int_0^{\cosh^{-1}x} \sec t \, dt \cdot \frac{d\cosh^{-1}x}{dx}$$

$$= -\sec(-\tanh^{-1}x) \cdot \frac{-1}{1-x^2} + \sec(\cosh^{-1}x)\frac{1}{\sqrt{x^2-1}}$$

$$= \frac{\sec(-\tanh^{-1}x)}{1-x^2} + \frac{\sec(\cosh^{-1}x)}{\sqrt{x^2-1}}$$

If f is continuous and $\displaystyle\int_1^{10} f(t)\,dt = 12$, find $\displaystyle\int_{-1}^{2} f(7-3t)\,dt$. **SCORE: ___ / 15 POINTS**

$u = 7-3t$: $t=2 \Rightarrow u=1$; $t=-1 \Rightarrow u=10$

$$\frac{du}{dt} = -3$$

$$-\frac{1}{3}du = dt$$

$$\int_{10}^{1} -\frac{1}{3} f(u)\,du = \frac{1}{3}\int_1^{10} f(u)\,du = \frac{1}{3}(12) = 4$$

State the Fundamental Theorem of Calculus Parts 1 and 2. **SCORE: ___ / 10 POINTS**

SEE PREVIOUS QUIZ KEYS

Evaluate the following integrals. Show all relevant work. **SCORE: ___ / 15 POINTS**

[a] $\displaystyle\int \frac{\sec^2(\sinh^{-1} t)}{\sqrt{1+t^2}}\, dt$

SEE 9:30 VERSION C KEY

[b] $\displaystyle\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{t^3-t}{2-\cos t}\, dt$

$$\frac{(-t)^3-(-t)}{2-\cos(-t)} = \frac{-t^3+t}{2-\cos t} = -\frac{t^3-t}{2-\cos t}$$

SINCE THE INTEGRAND IS ODD ON $\left[-\frac{\pi}{3}, \frac{\pi}{3}\right]$, THE INTEGRAL IS 0

[c] $\displaystyle\int_1^{16} \frac{(2+\sqrt[4]{t})^2}{t}\, dt$

$$= \int_1^{16} \frac{4+4t^{\frac{1}{4}}+t^{\frac{1}{2}}}{t}\, dt$$

$$= \int_1^{16} \left(\frac{4}{t}+4t^{-\frac{3}{4}}+t^{-\frac{1}{2}}\right) dt$$

$$= \left(4\ln|t| + 16t^{\frac{1}{4}} + 2t^{\frac{1}{2}}\right)\Big|_1^{16}$$

$$= (4\ln 16 + 16(2) + 2(4)) - (4\ln 1 + 16 + 2)$$

$$= 4\ln 16 + 22$$

[d] $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\sin 2t}{2-\sin^2 t}\, dt$

$u = 2-\sin^2 t$; $t=\frac{\pi}{2} \Rightarrow u=1$; $t=0 \Rightarrow u=2$

$$\frac{du}{dt} = (-2\sin t)\cos t$$

$$-du = \sin 2t\, dt$$

$$-\int_2^1 \frac{1}{u}\, du$$

$$= \int_1^2 \frac{1}{u}\, du$$

$$= \ln|u|\Big|_1^2$$

$$= \ln 2 - \ln 1$$

$$= \ln 2$$