Math 1B (7:30am – 8:20am)
Midterm 2 Version A
Tue May 25, 2010

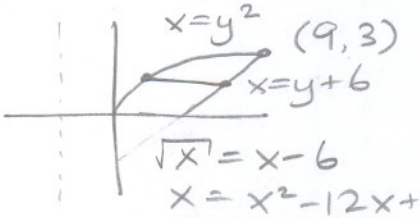
SCORE: ___ / 150 POINTS

What month is your birthday ? ___ ___
What are the first 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___
What are the last 2 digits of your social security number ? ___ ___
[IF YOU DO NOT HAVE A SOCIAL SECURITY NUMBER, USE YOUR STUDENT ID NUMBER]

NO CALCULATORS ALLOWED ON THIS SECTION
YOU MUST SHOW PROPER CALCULUS LEVEL WORK

Consider the region bounded by $y = \sqrt{x}$, $y = x - 6$ and $y = 0$. **SCORE: ___ / 30 POINTS**

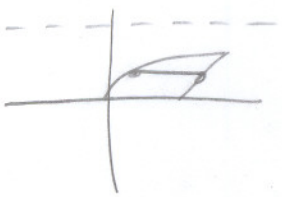
[a] If the region is revolved around $x = -4$, write, **BUT DO NOT EVALUATE**, an integral for the resulting volume.



$\sqrt{x} = x - 6$
$x = x^2 - 12x + 36$
$0 = x^2 - 13x + 36$
$0 = (x-9)(x-4) \Rightarrow x = 4, 9$

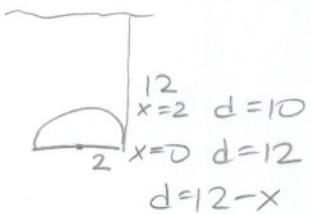
$$\int_0^3 \pi\left[(y+6-\text{-}4)^2 - (y^2-\text{-}4)^2\right] dy$$
$$= \int_0^3 \pi\left[(y+10)^2 - (y^2+4)^2\right] dy$$

[b] If the region is revolved around $y = 6$, write, **BUT DO NOT EVALUATE**, an integral for the resulting volume.



$$\int_0^3 2\pi(6-y)(y+6-y^2)\, dy$$

The window of an aquarium is a semicircle of radius 2 feet with its flat side down. If the flat side is 12 feet below the surface of the water, find the hydrostatic force on the window. **Use ρ as the density of water in your work.** **SCORE: ___ / 20 POINTS**



$d = 12 - x$

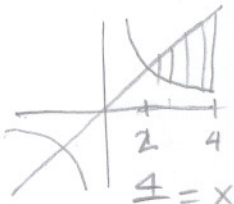
$$\int_0^2 \rho(12-x)\left(2\sqrt{4-x^2}\right) dx$$

$u = 4 - x^2$
$-\frac{1}{2}du = x\,dx$

$$= 2\rho\left[12\int_0^2 \sqrt{4-x^2}\, dx - \int_0^2 x\sqrt{4-x^2}\, dx\right]$$
$$= 2\rho\left[12 \cdot \tfrac{1}{4}\pi(2^2) - \int_4^0 -\tfrac{1}{2}u^{\frac{1}{2}}\, du\right]$$
$$= 2\rho\left[12\pi + \tfrac{1}{3}u^{\frac{3}{2}}\Big|_4^0\right]$$
$$= 2\rho\left[12\pi - \tfrac{8}{3}\right] \text{ ft-lb}$$

The base of a solid is the region between $y = \frac{4}{x}$ and $y = x$ on the interval $[2, 4]$. Cross sections perpendicular to the x–axis are isosceles right triangles with hypotenuse in the base. Write, **BUT DO NOT EVALUATE**, an integral for the volume of the solid.

SCORE: ___ / 10 POINTS



$\frac{4}{x} = x$

$4 = x^2$

$x = 2, -2$

$$\int_2^4 \frac{1}{4}\left(x - \frac{4}{x}\right)^2 dx$$

A 60 pound bucket is attached to a 100 pound chain hanging from the roof of a 50 foot building. The chain has constant density throughout its 50 foot length. Using the chain, you pull the bucket up the building to a window 10 feet from the roof, where someone removes the bucket from the chain. You then pull the remainder of the chain to the roof. Write, **BUT DO NOT EVALUATE**, an integral expression for your work done.

SCORE: ___ / 12 POINTS

$$\underbrace{\int_0^{50} 2x\,dx}_{\text{CHAIN TO ROOF}} + \underbrace{60(50-10)}_{\text{BUCKET}} \text{ ft-lb}$$

Find the area between the graphs of $f(x) = 2x^2$ and $g(x) = x^2 + 1$ on the interval $[0, 3]$.

SCORE: ___ / 15 POINTS

$2x^2 = x^2 + 1$

$x^2 = 1$

$x = 1, -1$

$$\int_0^1 (x^2 + 1 - 2x^2)\,dx + \int_1^3 (2x^2 - (x^2+1))\,dx$$

$$= \int_0^1 (1 - x^2)\,dx + \int_1^3 (x^2 - 1)\,dx$$

$$= \left(x - \frac{1}{3}x^3\right)\Big|_0^1 + \left(\frac{1}{3}x^3 - x\right)\Big|_1^3$$

$$= \left(1 - \frac{1}{3}\right) + (9 - 3) - \left(\frac{1}{3} - 1\right)$$

$$= 7\frac{1}{3}$$

Find the average value of $f(x) = 3x^2 - 2x$ on the interval $[-2, 1]$.

SCORE: ___ / 10 POINTS

$$\frac{1}{1 - -2}\int_{-2}^{1} (3x^2 - 2x)\,dx$$

$$= \frac{1}{3}\left(x^3 - x^2\right)\Big|_{-2}^{1}$$

$$= \frac{1}{3}\left(1 - 1 - (-8 - 4)\right)$$

$$= 4$$

Evaluate the following integrals.

SCORE: ___ / 30 POINTS

[a] $\int_{-1}^{1} \frac{2x(x-4)}{\sqrt[3]{3+6x^2-x^3}}\,dx$

$u = 3+6x^2-x^3$

$\frac{du}{dx} = 12x-3x^2 = 3x(4-x)$

$-\frac{2}{3}\,du = 2x(x-4)\,dx$

$\int_{10}^{8} -\frac{2}{3}u^{-\frac{1}{3}}\,du$

$= -u^{\frac{2}{3}}\Big|_{10}^{8}$

$= -4--10^{\frac{2}{3}}$

$= 10^{\frac{2}{3}}-4$

[b] $\int \frac{1}{(\sqrt{\cot^2 x-1})(\sin^2 x)}\,dx$

$u = \cot x$

$\frac{du}{dx} = -\csc^2 x = \frac{-1}{\sin^2 x}$

$-du = \frac{1}{\sin^2 x}\,dx$

$\int -\frac{1}{\sqrt{u^2-1}}\,du$

$= -\cosh^{-1}u + C$

$= -\cosh^{-1}\cot x + C$

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CALCULATORS ALLOWED ON THIS SECTION

Consider the graph of $y = \ln x$ from $y = 0$ to $y = 1$.

SCORE: __ / 23 POINTS

[a] Write, **BUT DO NOT EVALUATE**, a dx integral for the length of the curve.

$$\int_1^e \sqrt{1 + \frac{1}{x^2}}\, dx$$

[b] If the curve is revolved around the y–axis, write, **BUT DO NOT EVALUATE**, a dy integral for the resulting surface area.

$$\int_0^1 2\pi e^y \sqrt{1 + e^{2y}}\, dy$$

[c] Use your calculator's fnInt feature to estimate the surface area in [b] to 2 decimal places.

22.94