

What month is your birthday? _____

What are the first 2 digits of your address? _____

What are the last 2 digits of your zip code? _____

What are the last 2 digits of your social security number? _____

[IF YOU DO NOT HAVE A SOCIAL SECURITY NUMBER,
USE YOUR STUDENT ID NUMBER]

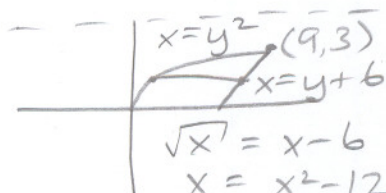
SCORE: ____ / 150 POINTS

NO CALCULATORS ALLOWED ON THIS SECTION YOU MUST SHOW PROPER CALCULUS LEVEL WORK

Consider the region bounded by $y = \sqrt{x}$, $y = x - 6$ and $y = 0$.

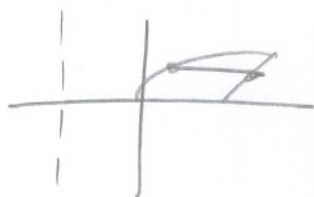
SCORE: ____ / 30 POINTS

- [a] If the region is revolved around $y = 4$, write, **BUT DO NOT EVALUATE**, an integral for the resulting volume.



$$\int_0^3 2\pi (4 - y)(y + 6 - y^2) dy$$

- [b] If the region is revolved around $x = -6$, write, **BUT DO NOT EVALUATE**, an integral for the resulting volume.



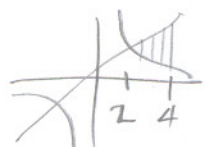
$$\int_0^3 \pi [(y + 6 - (-6))^2 - (y^2 - (-6))^2] dy$$

$$= \int_0^3 \pi [(y + 12)^2 - (y^2 + 6)^2] dy$$

The base of a solid is the region between $y = \frac{4}{x}$ and $y = x$ on the interval $[2, 4]$. Cross sections

SCORE: ____ / 10 POINTS

perpendicular to the x -axis are isosceles right triangles with hypotenuse in the base. Write, **BUT DO NOT EVALUATE**, an integral for the volume of the solid.



$$\int_2^4 \frac{1}{2} \left(x - \frac{4}{x}\right)^2 dx$$

$$\frac{4}{x} = x$$

$$4 = x^2$$

$$x = 2, -2$$

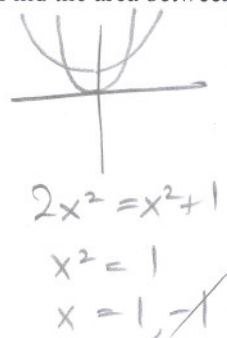
A 60 pound bucket is attached to a 100 pound chain hanging from the roof of a 50 foot building. The chain has constant density throughout its 50 foot length. Using the chain, you pull the bucket up the building to a window 10 feet from the roof, where someone removes the bucket from the chain. You then pull the remainder of the chain to the roof. Write, **BUT DO NOT EVALUATE**, an integral expression for your work done.

SCORE: ____ / 12 POINTS

$$\underbrace{\int_0^{50} 2x dx}_{\text{CHAIN TO ROOF}} + \underbrace{60(50 - 10)}_{\text{BUCKET}} \text{ ft-lb}$$

Find the area between the graphs of $f(x) = 2x^2$ and $g(x) = x^2 + 1$ on the interval $[0, 3]$.

SCORE: ___ / 15 POINTS



$$\begin{aligned}
 & \int_0^1 (x^2 + 1 - 2x^2) dx + \int_1^3 (2x^2 - (x^2 + 1)) dx \\
 &= \int_0^1 (1 - x^2) dx + \int_1^3 (x^2 - 1) dx \\
 &= \left(x - \frac{1}{3}x^3\right) \Big|_0^1 + \left(\frac{1}{3}x^3 - x\right) \Big|_1^3 \\
 &= \left(1 - \frac{1}{3}\right) + \left(9 - 3\right) - \left(\frac{1}{3} - 1\right) \\
 &= 7\frac{1}{3}
 \end{aligned}$$

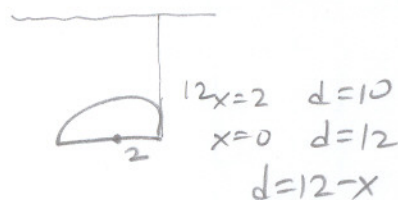
Find the average value of $f(x) = 3x^2 - 2x$ on the interval $[-2, 1]$.

SCORE: ___ / 10 POINTS

$$\begin{aligned}
 & \frac{1}{1 - (-2)} \int_{-2}^1 (3x^2 - 2x) dx \\
 &= \frac{1}{3} (x^3 - x^2) \Big|_{-2}^1 \\
 &= \frac{1}{3} (1 - 1 - (-8 - 4)) \\
 &= 4
 \end{aligned}$$

The window of an aquarium is a semicircle of radius 2 feet with its flat side down. If the flat side is 12 feet below the surface of the water, find the hydrostatic force on the window. Use ρ as the density of water in your work.

SCORE: ___ / 20 POINTS



$$\begin{aligned}
 & \int_0^2 \rho (12 - x) (2 \sqrt{4 - x^2}) dx \\
 &= 2\rho \left[12 \int_0^2 \sqrt{4 - x^2} dx - \int_0^2 x \sqrt{4 - x^2} dx \right] \\
 &= 2\rho \left[12 \cdot \frac{1}{4} \pi (2^2) - \int_4^0 -\frac{1}{2} u^{\frac{1}{2}} du \right] \quad \begin{matrix} \nwarrow u = 4 - x^2 \\ -\frac{1}{2} du = x dx \end{matrix} \\
 &= 2\rho \left[12\pi + \frac{1}{3} u^{\frac{3}{2}} \Big|_4^0 \right] \\
 &= 2\rho \left[12\pi - \frac{8}{3} \right] \text{ ft-lb}
 \end{aligned}$$

Evaluate the following integrals.

SCORE: 30 POINTS

[a] $\int \frac{1}{(\sqrt{\tan^2 x - 1})(\cos^2 x)} dx$

$$u = \tan x$$

$$\frac{du}{dx} = \sec^2 x = \frac{1}{\cos^2 x}$$

$$du = \frac{1}{\cos^2 x} dx$$

$$\int \frac{1}{\sqrt{u^2 - 1}} du$$

$$= \cosh^{-1} u + C$$

$$= \cosh^{-1} \tan x + C$$

[b] $\int_{-1}^1 \frac{2x(x-4)}{\sqrt[3]{3+6x^2-x^3}} dx$

$$u = 3 + 6x^2 - x^3$$

$$\frac{du}{dx} = 12x - 3x^2 = 3x(4-x)$$

$$-\frac{2}{3} du = 2x(x-4) dx$$

$$\int_{10}^8 -\frac{2}{3} u^{-\frac{1}{3}} du$$

$$= -u^{\frac{2}{3}} \Big|_{10}^8$$

$$= -4 - -10^{\frac{2}{3}}$$

$$= 10^{\frac{2}{3}} - 4$$

CALCULATORS ALLOWED ON THIS SECTION

Consider the graph of $y = \ln x$ from $y = 0$ to $y = 1$.

SCORE: ___ / 23 POINTS

- [a] Write, **BUT DO NOT EVALUATE**, a dx integral for the length of the curve.

$$\int_1^e \sqrt{1 + \frac{1}{x^2}} dx$$

- [b] If the curve is revolved around the y -axis, write, **BUT DO NOT EVALUATE**, a dy integral for the resulting surface area.

$$\int_0^1 2\pi e^y \sqrt{1 + e^{2y}} dy$$

- [c] Use your calculator's fnInt feature to estimate the surface area in [b] to 2 decimal places.

22.94