

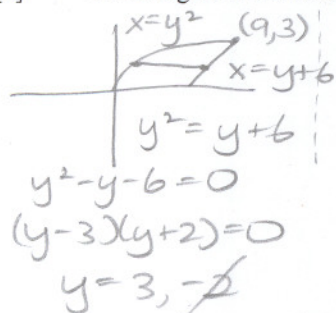
SCORE: \_\_\_\_ / 150 POINTS

## NO CALCULATORS ALLOWED ON THIS SECTION YOU MUST SHOW PROPER CALCULUS LEVEL WORK

Consider the region bounded by  $y = \sqrt{x}$ ,  $y = x - 6$  and  $y = 0$ .

SCORE: \_\_\_\_ / 30 POINTS

- [a] If the region is revolved around  $x = 10$ , write, **BUT DO NOT EVALUATE**, an integral for the resulting volume.



$$\int_0^3 \pi [(10 - y^2)^2 - (10 - (y + 6))^2] dy$$

$$= \int_0^3 \pi [(10 - y^2)^2 - (4 - y)^2] dy$$

- [b] If the region is revolved around  $y = -12$ , write, **BUT DO NOT EVALUATE**, an integral for the resulting volume.



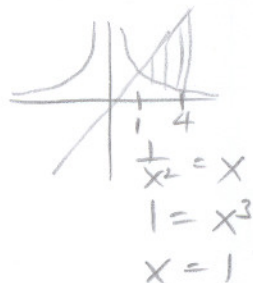
$$\int_0^3 2\pi (y - (-12))(y + 6 - y^2) dy$$

$$= \int_0^3 2\pi (y + 12)(6 + y - y^2) dy$$

The base of a solid is the region between  $y = \frac{1}{x^2}$  and  $y = x$  on the interval  $[1, 4]$ . Cross sections

SCORE: \_\_\_\_ / 10 POINTS

perpendicular to the  $x$ -axis are semicircles. Write, **BUT DO NOT EVALUATE**, an integral for the volume of the solid.



$$\int_1^4 \frac{\pi}{8} (x - \frac{1}{x^2})^2 dx$$

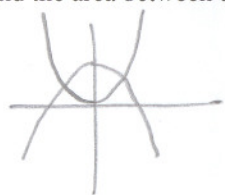
A 40 pound bucket is attached to a 60 pound chain hanging from the roof of a 30 foot building. The chain has constant density throughout its 30 foot length. Using the chain, you pull the bucket up the building to a window 10 feet from the roof, where someone removes the bucket from the chain. You then pull the remainder of the chain to the roof. Write, **BUT DO NOT EVALUATE**, an integral expression for your work done.

SCORE: \_\_\_\_ / 12 POINTS

$$\underbrace{\int_0^{30} 2x dx}_{\text{CHAIN TO ROOF}} + \underbrace{40(30 - 10) \text{ ft-lb}}_{\text{BUCKET}}$$

Find the area between the graphs of  $f(x) = x^2$  and  $g(x) = 2 - x^2$  on the interval  $[0, 3]$ .

SCORE: 15 POINTS



$$x^2 = 2 - x^2$$

$$2x^2 = 2$$

$$x^2 = 1$$

$$x = 1, \text{ and } x = -1$$

$$\begin{aligned} & \int_0^1 (2 - x^2 - x^2) dx + \int_1^3 (x^2 - (2 - x^2)) dx \\ &= \int_0^1 (2 - 2x^2) dx + \int_1^3 (2x^2 - 2) dx \\ &= (2x - \frac{2}{3}x^3) \Big|_0^1 + (\frac{2}{3}x^3 - 2x) \Big|_1^3 \\ &= (2 - \frac{2}{3}) + (18 - 6) - (\frac{2}{3} - 2) \\ &= 14\frac{2}{3} \end{aligned}$$

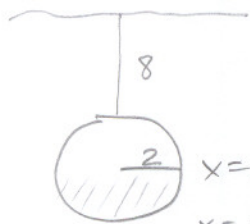
Find the average value of  $f(x) = 3x^2 - 2x$  on the interval  $[-1, 2]$ .

SCORE: 10 POINTS

$$\begin{aligned} & \frac{1}{2 - (-1)} \int_{-1}^2 (3x^2 - 2x) dx \\ &= \frac{1}{3} (x^3 - x^2) \Big|_{-1}^2 \\ &= \frac{1}{3} (8 - 4 - (-1 - 1)) \\ &= 2 \end{aligned}$$

A spherical tank of radius 2 feet containing water is buried underground. The top of the tank is 8 feet below ground level. Find the work done in pumping the water to ground level if the tank is half full. Use  $\rho$  as the density of water in your work.

SCORE: 20 POINTS



$$\begin{aligned} x = 0 \quad d = 10 \\ x = 2 \quad d = 12 \\ d = x + 10 \end{aligned}$$

$$\begin{aligned} & \int_0^2 \rho \pi (\sqrt{4 - x^2})^2 (x + 10) dx \\ &= \rho \pi \int_0^2 (4 - x^2)(x + 10) dx \\ &= \rho \pi \int_0^2 (40 + 4x - 10x^2 - x^3) dx \\ &= \rho \pi (40x + 2x^2 - \frac{10}{3}x^3 - \frac{1}{4}x^4) \Big|_0^2 \\ &= \rho \pi (80 + 8 - \frac{80}{3} - 4) \\ &= \rho \pi (84 - 26\frac{2}{3}) \\ &= \frac{172\rho\pi}{3} \text{ ft-lb} \end{aligned}$$



[a]  $\int \frac{1}{(\sqrt{\cot^2 x - 1})(\sin^2 x)} dx$

$$u = \cot x$$

$$\frac{du}{dx} = -\csc^2 x = -\frac{1}{\sin^2 x}$$

$$-du = \frac{1}{\sin^2 x} dx$$

$$\int \frac{1}{\sqrt{u^2 - 1}} du$$

$$= -\cosh^{-1} u + C$$

$$= -\cosh^{-1} \cot x + C$$

[b]  $\int_{-1}^1 \frac{2x(x-3)}{\sqrt[3]{1+9x^2-2x^3}} dx$

$$u = 1 + 9x^2 - 2x^3$$

$$\frac{du}{dx} = 18x - 6x^2 = 6x(3-x)$$

$$-\frac{1}{3} du = 2x(x-3) dx$$

$$\int_{12}^8 -\frac{1}{3} u^{-\frac{1}{3}} du$$

$$= -\frac{1}{2} u^{\frac{2}{3}} \Big|_{12}^8$$

$$= -\frac{1}{2} (4 - 12^{\frac{2}{3}})$$

$$= \frac{1}{2} (12^{\frac{2}{3}} - 4)$$

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## CALCULATORS ALLOWED ON THIS SECTION

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Consider the graph of  $y = \arctan x$  from  $y = 0$  to  $y = 1$ .

SCORE: \_\_\_/23 POINTS

- [a] Write, **BUT DO NOT EVALUATE**, a  $dx$  integral for the length of the curve.

$$\int_0^{\tan 1} \sqrt{1 + \left(\frac{1}{1+x^2}\right)^2} dx$$

- [b] If the curve is revolved around the  $y$ -axis, write, **BUT DO NOT EVALUATE**, a  $dy$  integral for the resulting surface area.

$$\int_0^1 2\pi \tan y \sqrt{1 + \sec^4 y} dy$$

- [c] Use your calculator's fnInt feature to estimate the surface area in [b] to 2 decimal places.

8.63