Math 1B (9:30am – 10:20am)
Midterm 2 Version D
Tue May 25, 2010

SCORE: ___ / 150 POINTS

What month is your birthday ? ___ ___
What are the first 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___
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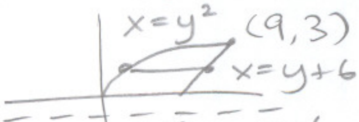
NO CALCULATORS ALLOWED ON THIS SECTION
YOU MUST SHOW PROPER CALCULUS LEVEL WORK

Consider the region bounded by $y=\sqrt{x}$, $y=x-6$ and $y=0$. **SCORE: ___ / 30 POINTS**

[a] If the region is revolved around $y=-10$, write, **BUT DO NOT EVALUATE**, an integral for the resulting volume.



$y^2=y+6$

$y^2-y-6=0$

$(y-3)(y+2)=0$

$y=3, \cancel{-2}$

$$\int_0^3 2\pi(y--10)(y+6-y^2)\,dy$$

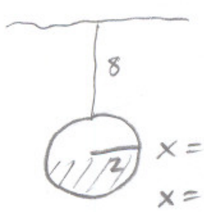
$$=\int_0^3 2\pi(y+10)(6+y-y^2)\,dy$$

[b] If the region is revolved around $x=12$, write, **BUT DO NOT EVALUATE**, an integral for the resulting volume.

$$\int_0^3 \pi\left[(12-y^2)^2-(12-(y+6))^2\right]dy$$

$$=\int_0^3 \pi\left[(12-y^2)^2-(6-y)^2\right]dy$$

A spherical tank of radius 2 feet containing water is buried underground. The top of the tank is 8 feet below ground level. Find the work done in pumping the water to ground level if the tank is half full. **Use ρ as the density of water in your work.** **SCORE: ___ / 20 POINTS**



$x=0 \quad d=10$

$x=2 \quad d=12$

$d=x+10$

$$\int_0^2 \rho\pi\left(\sqrt{4-x^2}\right)^2(x+10)\,dx$$

$$=\rho\pi\int_0^2(4-x^2)(x+10)\,dx$$

$$=\rho\pi\int_0^2(40+4x-10x^2-x^3)\Big|_0^2$$

$$=\rho\pi\left(40x+2x^2-\tfrac{10}{3}x^3-\tfrac{1}{4}x^4\right)\Big|_0^2$$

$$=\rho\pi\left(80+8-\tfrac{80}{3}-4\right)$$

$$=\rho\pi\left(84-\tfrac{80}{3}\right)$$

$$=\frac{172\rho\pi}{3}\text{ ft-lb}$$

The base of a solid is the region between $y = \dfrac{1}{x^2}$ and $y = x$ on the interval $[1, 4]$. Cross sections perpendicular to the x–axis are semicircles. Write, **BUT DO NOT EVALUATE**, an integral for the volume of the solid. SCORE: ___ / 10 POINTS

$$\int_1^4 \frac{\pi}{8}\left(x - \frac{1}{x^2}\right)^2 dx$$

$\frac{1}{x^2} = x$

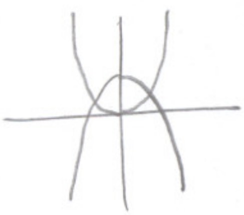
$1 = x^3$

$x = 1$

A 40 pound bucket is attached to a 60 pound chain hanging from the roof of a 30 foot building. The chain has constant density throughout its 30 foot length. Using the chain, you pull the bucket up the building to a window 10 feet from the roof, where someone removes the bucket from the chain. You then pull the remainder of the chain to the roof. Write, **BUT DO NOT EVALUATE**, an integral expression for your work done. SCORE: ___ / 12 POINTS

$$\underbrace{\int_0^{30} 2x\,dx}_{\text{CHAIN TO ROOF}} + \underbrace{40(30-10)}_{\text{BUCKET}} \text{ ft-lb}$$

Find the area between the graphs of $f(x) = x^2$ and $g(x) = 2 - x^2$ on the interval $[0, 3]$. SCORE: ___ / 15 POINTS



$x^2 = 2 - x^2$

$2x^2 = 2$

$x^2 = 1$

$x = 1, \cancel{-1}$

$$\int_0^1 (2 - x^2 - x^2)\,dx + \int_1^3 (x^2 - (2 - x^2))\,dx$$

$$= \int_0^1 (2 - 2x^2)\,dx + \int_1^3 (2x^2 - 2)\,dx$$

$$= \left(2x - \tfrac{2}{3}x^3\right)\Big|_0^1 + \left(\tfrac{2}{3}x^3 - 2x\right)\Big|_1^3$$

$$= \left(2 - \tfrac{2}{3}\right) + (18 - 6) - \left(\tfrac{2}{3} - 2\right)$$

$$= 14\tfrac{2}{3}$$

Find the average value of $f(x) = 3x^2 - 2x$ on the interval $[-1, 2]$. SCORE: ___ / 10 POINTS

$$\frac{1}{2 - -1} \int_{-1}^{2} (3x^2 - 2x)\,dx$$

$$= \tfrac{1}{3}\left(x^3 - x^2\right)\Big|_{-1}^{2}$$

$$= \tfrac{1}{3}\left(8 - 4 - (-1 - 1)\right)$$

$$= 2$$

Evaluate the following integrals. SCORE: ___ / 30 POINTS

[a] $$\int_{-1}^{1} \frac{2x(x-3)}{\sqrt[3]{1+9x^2-2x^3}}\, dx$$

$$u = 1+9x^2-2x^3$$

$$\frac{du}{dx} = 18x-6x^2 = 6x(3-x)$$

$$-\frac{1}{3}du = 2x(x-3)dx$$

$$\int_{12}^{8} -\frac{1}{3}u^{-\frac{1}{3}}du$$

$$= -\frac{1}{2}u^{\frac{2}{3}}\Big|_{12}^{8}$$

$$= -\frac{1}{2}(4-12^{\frac{2}{3}})$$

$$= \frac{1}{2}(12^{\frac{2}{3}}-4)$$

[b] $$\int \frac{1}{(\sqrt{\tan^2 x-1})(\cos^2 x)}\, dx$$

$$u = \tan x$$

$$\frac{du}{dx} = \sec^2 x = \frac{1}{\cos^2 x}$$

$$du = \frac{1}{\cos^2 x}dx$$

$$\int \frac{1}{\sqrt{u^2-1}}du$$

$$= \cosh^{-1} u + C$$

$$= \cosh^{-1} \tan x + C$$

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CALCULATORS ALLOWED ON THIS SECTION

Consider the graph of $y = \arctan x$ from $y = 0$ to $y = 1$.

SCORE: ___ / 23 POINTS

[a] Write, **BUT DO NOT EVALUATE**, a dx integral for the length of the curve.

$$\int_0^{\tan 1} \sqrt{1 + \left(\frac{1}{1+x^2}\right)^2}\, dx$$

[b] If the curve is revolved around the y–axis, write, **BUT DO NOT EVALUATE**, a dy integral for the resulting surface area.

$$\int_0^1 2\pi \tan y \sqrt{1 + \sec^4 y}\, dy$$

[c] Use your calculator's fnInt feature to estimate the surface area in [b] to 2 decimal places.

8.63