

What month is your birthday? _____

What are the first 2 digits of your address? _____

What are the last 2 digits of your zip code? _____

What are the last 2 digits of your social security number? _____

[IF YOU DO NOT HAVE A SOCIAL SECURITY NUMBER,
USE YOUR STUDENT ID NUMBER]

SCORE: ____ / 150 POINTS

You may or may not need the following reduction formulae.

$$\int \sin^n u \, du = -\frac{1}{n} \sin^{n-1} u \cos u + \frac{n-1}{n} \int \sin^{n-2} u \, du \quad \text{if } n \neq 0$$

$$\int \cos^n u \, du = \frac{1}{n} \cos^{n-1} u \sin u + \frac{n-1}{n} \int \cos^{n-2} u \, du \quad \text{if } n \neq 0$$

$$\int \sec^n u \, du = \frac{1}{n-1} \sec^{n-2} u \tan u + \frac{n-2}{n-1} \int \sec^{n-2} u \, du \quad \text{if } n \neq 1$$

NO CALCULATORS**YOU MUST SHOW PROPER CALCULUS LEVEL WORK**Determine if $\int_0^1 \frac{5 - \cos x}{x^2} dx$ converges or diverges.

SCORE: ____ / ____ POINTS

$$\begin{aligned} \cos x &\leq 1 \\ -\cos x &\geq -1 \\ 5 - \cos x &\geq 4 \end{aligned}$$

$$\frac{5 - \cos x}{x^2} \geq \frac{4}{x^2} > 0$$

$$\int_0^1 \frac{4}{x^2} dx = 4 \int_0^1 \frac{1}{x^2} dx \text{ DIVERGES SINCE } p=2 \geq 1$$

$$\text{so } \int_0^1 \frac{5 - \cos x}{x^2} dx \text{ DIVERGES}$$

Find $\int_0^{\pi/4} \tan^2 x \sec^6 x \, dx$.

SCORE: ____ / ____ POINTS

$$u = \tan x \begin{cases} x = \pi/4 \rightarrow u = 1 \\ x = 0 \rightarrow u = 0 \end{cases}$$

$$du = \sec^2 x \, dx$$

$$\int_0^1 u^2 (u^2 + 1)^2 du$$

$$= \int_0^1 u^2 (u^4 + 2u^2 + 1) du$$

$$= \int_0^1 (u^6 + 2u^4 + u^2) du$$

$$= \left(\frac{1}{7} u^7 + \frac{2}{5} u^5 + \frac{1}{3} u^3 \right) \Big|_0^1$$

$$= \frac{1}{7} + \frac{2}{5} + \frac{1}{3} = \frac{92}{105}$$

Find $\int \frac{5-3x^2}{x^3-2x^2+5x} dx$.

SCORE: ___ / ___ POINTS

$$= \int \frac{5-3x^2}{x(x^2-2x+5)} dx$$

$$\frac{5-3x^2}{x(x^2-2x+5)} = \frac{A}{x} + \frac{B(2x-2)+C}{x^2-2x+5}$$

$$= \int \left(\frac{1}{x} + \frac{-2(2x-2)}{x^2-2x+5} + \frac{-2}{(x-1)^2+4} \right) dx$$

$$5-3x^2 = A(x^2-2x+5) + Bx(2x-2) + Cx$$

$$= \ln|x| - 2\ln(x^2-2x+5)$$

$$x=0$$

$$5 = 5A \rightarrow A=1$$

$$x=1$$

$$2 = 4A + C \rightarrow C = 2 - 4A = -2$$

$$x=2$$

$$-7 = 5A + 4B + 2C$$

$$-7 = 1 + 4B \rightarrow B = -2$$

$$- \tan^{-1} \frac{x-1}{2} + C$$

CHECK: $x = -1$

$$\frac{2}{-8} = -\frac{1}{4}$$

$$\frac{1}{-1} + \frac{(-2)(-4) + (-2)}{8} = -1 + \frac{3}{4} = -\frac{1}{4}$$

Find $\int_0^{\frac{1}{2}} \frac{1}{x(\ln x)^3} dx$.

SCORE: ___ / ___ POINTS

$$= \lim_{N \rightarrow 0^+} \int_N^{\frac{1}{2}} \frac{1}{x(\ln x)^3} dx$$

$$u = \ln x$$

$$du = \frac{1}{x} dx$$

$$= \lim_{N \rightarrow 0^+} -\frac{1}{2} \frac{1}{(\ln x)^2} \Big|_N^{\frac{1}{2}}$$

$$\int \frac{1}{u^3} du = -\frac{1}{2} u^{-2} = -\frac{1}{2} \frac{1}{(\ln x)^2}$$

$$= \lim_{N \rightarrow 0^+} -\frac{1}{2} \left(\frac{1}{(\ln \frac{1}{2})^2} - \frac{1}{(\ln N)^2} \right)$$

As $N \rightarrow 0^+$

$$\ln N \rightarrow -\infty$$

$$(\ln N)^2 \rightarrow \infty$$

$$\frac{1}{(\ln N)^2} \rightarrow 0$$

$$= -\frac{1}{2} \frac{1}{(\ln \frac{1}{2})^2}$$

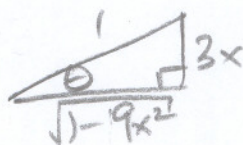
OR $-\frac{1}{2(\ln 2)^2}$

Find $\int \frac{\sqrt{1-9x^2}}{x^2} dx$.

$$x = \frac{1}{3} \sin \theta \rightarrow \sin \theta = 3x$$
$$dx = \frac{1}{3} \cos \theta d\theta$$

SCORE: ____ / ____ **POINTS**

$$= \int \frac{\cos \theta}{\frac{1}{9} \sin^2 \theta} \cdot \frac{1}{3} \cos \theta d\theta$$



$$= 3 \int \frac{\cos^2 \theta}{\sin^2 \theta} d\theta$$

$$= 3 \int \cos^2 \theta d\theta$$

$$= 3 \int (\csc^2 \theta - 1) d\theta$$

$$= 3(-\cot \theta - \theta) + C$$

$$= -3 \left(\frac{\sqrt{1-9x^2}}{3x} + \sin^{-1} 3x \right) + C$$

$$= \frac{-\sqrt{1-9x^2}}{x} - 3 \sin^{-1} 3x + C$$

Find $\int_1^{\infty} \frac{\ln x}{x^3} dx$.

SCORE: ____ / ____ POINTS

$$= \lim_{N \rightarrow \infty} \int_1^N \frac{\ln x}{x^3} dx$$

$$= \lim_{N \rightarrow \infty} \left(-\frac{1}{2} x^{-2} \ln x - \frac{1}{4} x^{-2} \right) \Big|_1^N$$

$$= \lim_{N \rightarrow \infty} \left(-\frac{1}{2} \frac{\ln N}{N^2} - \frac{1}{4N^2} - \left(0 - \frac{1}{4}\right) \right)$$

$$= 0 - 0 + \frac{1}{4}$$

$$= \frac{1}{4}$$

$$\begin{array}{r} \frac{v}{\ln x} + \frac{du}{x^{-3}} \\ \frac{1}{x} - \frac{1}{2}x^{-2} \\ \hline x - \frac{1}{2}x^{-3} \\ \hline 0 + \frac{1}{4}x^{-2} \end{array}$$

$$\lim_{N \rightarrow \infty} \frac{\ln N}{N^2} = \frac{\infty}{\infty}$$

$$= \lim_{N \rightarrow \infty} \frac{\frac{1}{N}}{2N} \quad \frac{0}{\infty}$$

$$= 0$$

Find $\int 3x^2 \tan^{-1} x \, dx$.

SCORE: ___ / ___ POINTS

$$= x^3 \tan^{-1} x - \int \frac{x^3}{1+x^2} dx$$

$$= x^3 \tan^{-1} x - \int \left(x - \frac{x}{1+x^2} \right) dx$$

$$= x^3 \tan^{-1} x - \frac{1}{2}x^2 + \frac{1}{2}\ln(1+x^2) + C$$

$$\begin{array}{l} u \quad dv \\ \tan^{-1} x \quad 3x^2 \\ \frac{1}{1+x^2} \quad \Rightarrow x^3 \end{array}$$

$$\begin{array}{r} x^2+1 \overline{) \begin{array}{r} x^3 \\ x^3+x \\ \hline -x \end{array}} \end{array}$$

BONUS QUESTION

Find [REDACTED].

SCORE: ___ / ___ POINTS

NOTE: This question will require very organized work. In fact, the right organization will greatly simplify the work.