

SCORE: ____ / 150 POINTS

You may or may not need the following reduction formulae.

$$\int \sin^n u \, du = -\frac{1}{n} \sin^{n-1} u \cos u + \frac{n-1}{n} \int \sin^{n-2} u \, du \quad \text{if } n \neq 0$$

$$\int \cos^n u \, du = \frac{1}{n} \cos^{n-1} u \sin u + \frac{n-1}{n} \int \cos^{n-2} u \, du \quad \text{if } n \neq 0$$

$$\int \sec^n u \, du = \frac{1}{n-1} \sec^{n-2} u \tan u + \frac{n-2}{n-1} \int \sec^{n-2} u \, du \quad \text{if } n \neq 1$$

NO CALCULATORS
YOU MUST SHOW PROPER CALCULUS LEVEL WORK

Find $\int 3x^2 \tan^{-1} x \, dx$.

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SEE 7:30 VERSION A QUESTION 7

Find $\int_0^{\pi/3} \tan^3 x \sec^3 x \, dx$.

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$$u = \sec x \begin{cases} x = \pi/3 \rightarrow u = 2 \\ x = 0 \rightarrow u = 1 \end{cases}$$

$$du = \sec x \tan x \, dx$$

$$\begin{aligned} & \int_1^2 (u^2 - 1) u^2 \, du \\ &= \int_1^2 (u^4 - u^2) \, du \\ &= \left(\frac{1}{5} u^5 - \frac{1}{3} u^3 \right) \Big|_1^2 \\ &= \frac{1}{5} (32 - 1) - \frac{1}{3} (8 - 1) \\ &= \frac{31}{5} - \frac{7}{3} \\ &= \frac{58}{15} \end{aligned}$$

Find $\int_1^{\infty} \frac{\ln x}{x^3} dx$.

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SEE 7:30 VERSION A QUESTION 6

Find $\int \frac{\sqrt{1-4x^2}}{x^2} dx$.

$x = \frac{1}{2} \sin \theta \rightarrow \sin \theta = 2x$
 $dx = \frac{1}{2} \cos \theta d\theta$

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$$= \int \frac{\cos \theta}{\frac{1}{4} \sin^2 \theta} \cdot \frac{1}{2} \cos \theta d\theta$$

$$= 2 \int \frac{\cos^2 \theta}{\sin^2 \theta} d\theta$$

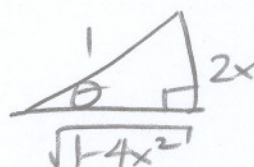
$$= 2 \int \cot^2 \theta d\theta$$

$$= 2 \int (\csc^2 \theta - 1) d\theta$$

$$= 2(-\cot \theta - \theta) + C$$

$$= -2 \left(\frac{\sqrt{1-4x^2}}{2x} + \sin^{-1} 2x \right) + C$$

$$= -\frac{\sqrt{1-4x^2}}{x} - 2 \sin^{-1} 2x + C$$



Find $\int_0^{\frac{1}{2}} \frac{1}{x(\ln x)^3} dx$.

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SEE 7:30 VERSION A QUESTION 4

Find $\int \frac{8-3x^2}{x^3+4x^2+8x} dx$.

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$$= \int \frac{8-3x^2}{x(x^2+4x+8)} dx$$

$$= \int \left(\frac{1}{x} + \frac{-2(2x+4)}{x^2+4x+8} + \frac{4}{(x+2)^2+4} \right) dx$$

$$= \ln|x| - 2 \ln(x^2+4x+8) + 2 \tan^{-1} \frac{x+2}{2} + C$$

$$\frac{8-3x^2}{x(x^2+4x+8)} = \frac{A}{x} + \frac{B(2x+4)+C}{x^2+4x+8}$$

$$8-3x^2 = A(x^2+4x+8) + Bx(2x+4) + Cx$$

$$x=0 \quad 8 = 8A \rightarrow A=1$$

$$x=-2 \quad -4 = 4A - 2C \rightarrow C = 2A + 2 = 4$$

$$x=1 \quad 5 = 13A + 6B + C$$

$$5 = 17 + 6B \rightarrow B = -2$$

CHECK: $x = -1$

$$\frac{5}{-5} = -1$$

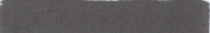
$$\frac{1}{-1} + \frac{(-2)(2)+4}{5} = -1$$

Determine if $\int_0^1 \frac{5 - \cos x}{x^2} dx$ converges or diverges.

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SEE 7:30 VERSION A QUESTION 1

BONUS QUESTION

Find 

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NOTE: This question will require very organized work. In fact, the right organization will greatly simplify the work.