Math 1B (9:30am – 10:20am)
Midterm 3 Version C
Tue Jun 15, 2010

SCORE: ___ / 150 POINTS

What month is your birthday ? ___ ___
What are the first 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___
What are the last 2 digits of your social security number ? ___ ___
[IF YOU DO NOT HAVE A SOCIAL SECURITY NUMBER, USE YOUR STUDENT ID NUMBER]

You may or may not need the following reduction formulae.

$$\int \sin^n u\, du = -\frac{1}{n}\sin^{n-1} u \cos u + \frac{n-1}{n}\int \sin^{n-2} u\, du \qquad \text{if } n \neq 0$$

$$\int \cos^n u\, du = \frac{1}{n}\cos^{n-1} u \sin u + \frac{n-1}{n}\int \cos^{n-2} u\, du \qquad \text{if } n \neq 0$$

$$\int \sec^n u\, du = \frac{1}{n-1}\sec^{n-2} u \tan u + \frac{n-2}{n-1}\int \sec^{n-2} u\, du \qquad \text{if } n \neq 1$$

NO CALCULATORS
YOU MUST SHOW PROPER CALCULUS LEVEL WORK

Determine if $\int_0^1 \frac{4-\sin x}{\sqrt{x}}\,dx$ converges or diverges.

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$\sin x \geq -1$

$-\sin x \leq 1$

$4 - \sin x \leq 5$

$0 < \frac{4-\sin x}{\sqrt{x}} \leq \frac{5}{\sqrt{x}}$

$\int_0^1 \frac{5}{\sqrt{x}}\,dx = 5\int_0^1 \frac{1}{\sqrt{x}}\,dx$ CONVERGES SINCE $p = \frac{1}{2} < 1$

SO $\int_0^1 \frac{4-\sin x}{\sqrt{x}}\,dx$ CONVERGES

Find $\int_0^{\frac{\pi}{3}} \tan^3 x \sec^3 x\, dx$.

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SEE 7:30 VERSION B QUESTION 2

Find $\int \frac{8-3x^2}{x^3-4x^2+8x}\,dx$.

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$$= \int \frac{8-3x^2}{x(x^2-4x+8)}\,dx$$

$$= \int \left(\frac{1}{x} + \frac{(-2)(2x-4)}{x^2-4x+8} + \frac{-4}{(x-2)^2+4}\right)dx$$

$$= \ln|x| - 2\ln(x^2-4x+8) - 2\tan^{-1}\frac{x-2}{2} + C$$

$$\frac{8-3x^2}{x(x^2-4x+8)} = \frac{A}{x} + \frac{B(2x-4)+C}{x^2-4x+8}$$

$$8-3x^2 = A(x^2-4x+8) + Bx(2x-4) + Cx$$

$x=0$ $\quad 8 = 8A \rightarrow A = 1$

$x=2$ $\quad -4 = 4A + 2C \rightarrow C = -2A - 2 = -4$

$x=1$ $\quad 5 = 5A - 2B + C$

$5 = 1 - 2B \rightarrow B = -2$

CHECK: $x = -1$ $\quad \frac{5}{-13}$

$$\frac{1}{-1} + \frac{(-2)(-6)-4}{13} = -1 + \frac{8}{13} = \frac{-5}{13}$$

Find $\int_0^1 \frac{\ln x}{\sqrt{x}}\,dx$.

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$$= \lim_{N\to 0^+} \int_N^1 \frac{\ln x}{\sqrt{x}}\,dx$$

$$= \lim_{N\to 0^+} \left(2\sqrt{x}\ln x - 4\sqrt{x}\right)\Big|_N^1$$

$$= \lim_{N\to 0^+} \left(2\sqrt{N}\ln N - 4\sqrt{N} - (0-4)\right)$$

$$= 0 - 0 + 4$$

$$= 4$$

u	dv
$\ln x$	$+\ x^{-\frac{1}{2}}$
$\frac{1}{x}$	$2x^{\frac{1}{2}}$
$-$	$-$
1	$2x^{-\frac{1}{2}}$
0	$4x^{\frac{1}{2}}$

$\times -\frac{1}{x}$

$$\lim_{N\to 0^+} \sqrt{N}\ln N \qquad 0\cdot -\infty$$

$$= \lim_{N\to 0^+} \frac{\ln N}{N^{-\frac{1}{2}}} \qquad \frac{-\infty}{\infty}$$

$$= \lim_{N\to 0^+} \frac{\frac{1}{N}}{-\frac{1}{2}N^{-\frac{3}{2}}}$$

$$= \lim_{N\to 0^+} -2N^{\frac{1}{2}}$$

$$= 0$$

Find $\int \frac{1}{x^2\sqrt{9x^2+1}}\,dx$.

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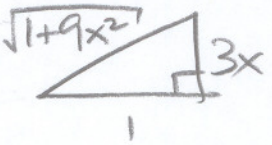
$x = \frac{1}{3}\tan\theta \rightarrow \tan\theta = 3x$

$dx = \frac{1}{3}\sec^2\theta$



$$= \int \frac{1}{\frac{1}{9}\tan^2\theta \sec\theta}\frac{1}{3}\sec^2\theta\,d\theta$$

$$= 3\int \frac{\sec\theta}{\tan^2\theta}\,d\theta$$

$$= 3\int \frac{\frac{1}{\cos\theta}}{\frac{\sin^2\theta}{\cos^2\theta}}\,d\theta$$

$$= 3\int \frac{\cos\theta}{\sin^2\theta}\,d\theta$$

$$= 3\int \csc\theta\cot\theta\,d\theta$$

$$= -3\csc\theta + C$$

$$= -3\left(\frac{\sqrt{1+9x^2}}{3x}\right) + C = -\frac{\sqrt{1+9x^2}}{x} + C$$

Find $\int_2^\infty \frac{1}{x(\ln x)^4}\,dx$.

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$$= \lim_{N\to\infty} \int_2^N \frac{1}{x(\ln x)^4}\,dx$$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$\int \frac{1}{u^4}\,du = -\frac{1}{3}u^{-3} = -\frac{1}{3}\frac{1}{(\ln x)^3}$

$$= \lim_{N\to\infty} -\frac{1}{3}\frac{1}{(\ln x)^3}\Big|_2^N$$

$$= \lim_{N\to\infty} -\frac{1}{3}\left(\frac{1}{(\ln N)^3} - \frac{1}{(\ln 2)^3}\right)$$

↳ As $N\to\infty$

$\ln N \to \infty$

$(\ln N)^3 \to \infty$

$\frac{1}{(\ln N)^3} \to 0$

$$= -\frac{1}{3}\left(0 - \frac{1}{(\ln 2)^3}\right)$$

$$= \frac{1}{3(\ln 2)^3}$$

Find $\int 3x^2 \tan^{-1} x \, dx$.

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SEE 7:30 VERSION A QUESTION 7

BONUS QUESTION

Find [redacted].

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NOTE: This question will require very organized work. In fact, the right organization will greatly simplify the work.