

SCORE: ____ / 150 POINTS

You may or may not need the following reduction formulae.

$$\int \sin^n u \, du = -\frac{1}{n} \sin^{n-1} u \cos u + \frac{n-1}{n} \int \sin^{n-2} u \, du \quad \text{if } n \neq 0$$

$$\int \cos^n u \, du = \frac{1}{n} \cos^{n-1} u \sin u + \frac{n-1}{n} \int \cos^{n-2} u \, du \quad \text{if } n \neq 0$$

$$\int \sec^n u \, du = \frac{1}{n-1} \sec^{n-2} u \tan u + \frac{n-2}{n-1} \int \sec^{n-2} u \, du \quad \text{if } n \neq 1$$

NO CALCULATORS
YOU MUST SHOW PROPER CALCULUS LEVEL WORK

Find $\int 3x^2 \tan^{-1} x \, dx$.

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SEE 7:30 VERSION A QUESTION 7

Find $\int_0^{\frac{\pi}{4}} \tan^2 x \sec^6 x \, dx$.

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SEE 7:30 VERSION A QUESTION 2

Find $\int_2^{\infty} \frac{1}{x(\ln x)^4} dx$.

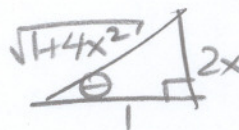
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SEE 9:30 VERSION C QUESTION 6

Find $\int \frac{1}{x^2 \sqrt{4x^2 + 1}} dx$.

$x = \frac{1}{2} \tan \theta \rightarrow \tan \theta = 2x$
 $dx = \frac{1}{2} \sec^2 \theta$

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$$= \int \frac{1}{\frac{1}{4} \tan^2 \theta \sec \theta} \cdot \frac{1}{2} \sec^2 \theta d\theta$$

$$= 2 \int \frac{\sec \theta}{\tan^2 \theta} d\theta$$

$$= 2 \int \frac{\frac{1}{\cos \theta}}{\frac{\sin^2 \theta}{\cos^2 \theta}} d\theta$$

$$= 2 \int \frac{\cos \theta}{\sin^2 \theta} d\theta$$

$$= 2 \int \csc \theta \cot \theta d\theta$$

$$= -2 \csc \theta + C$$

$$= -2 \left(\frac{\sqrt{1+4x^2}}{2x} \right) + C = -\frac{\sqrt{1+4x^2}}{x} + C$$

Find $\int_0^1 \frac{\ln x}{\sqrt{x}} dx$.

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SEE 9:30 VERSION C QUESTION 4

Find $\int \frac{13-3x^2}{x^3+6x^2+13x} dx$.

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$$= \int \frac{13-3x^2}{x(x^2+6x+13)} dx$$

$$\frac{13-3x^2}{x(x^2+6x+13)} = \frac{A}{x} + \frac{B(2x+6)+C}{x^2+6x+13}$$

$$= \int \left(\frac{1}{x} + \frac{(-2)(2x+6)}{x^2+6x+13} + \frac{6}{(x+3)^2+4} \right) dx$$

$$13-3x^2 = A(x^2+6x+13) + Bx(2x+6) + Cx$$

$$x=0 \quad 13 = 13A \rightarrow A=1$$

$$x=-3 \quad -14 = 4A - 3C \rightarrow C = \frac{4A+14}{3} = 6$$

$$x=1 \quad 10 = 20A + 8B + C$$

$$10 = 26 + 8B \rightarrow B = -2$$

$$= \ln|x| - 2 \ln(x^2+6x+13)$$

$$+ 3 \tan^{-1} \frac{x+3}{2}$$

$$+ C$$

CHECK: $x = -1$

$$\frac{10}{-8} = -\frac{5}{4}$$

$$\frac{1}{-1} + \frac{(-2)(4)+6}{8} = -1 - \frac{1}{4} = -\frac{5}{4}$$

Determine if $\int_0^1 \frac{4 - \sin x}{\sqrt{x}} dx$ converges or diverges.

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SEE 9:30 VERSION C QUESTION 1

BONUS QUESTION

Find .

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NOTE: This question will require very organized work. In fact, the right organization will greatly simplify the work.