Math 1B (7:30am – 8:20am)
Quiz 1 Version A
Fri Apr 9, 2010

SCORE: ___ / 30 POINTS

What month is your birthday ? ___ ___
What are the first 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___
What are the last 2 digits of your social security number ? ___ ___
[IF YOU DO NOT HAVE A SOCIAL SECURITY NUMBER, USE YOUR STUDENT ID NUMBER]

NO CALCULATORS ALLOWED
YOU MUST SHOW PROPER WORK (EXCEPT MULTIPLE CHOICE)

[MULTIPLE CHOICE] $\operatorname{sech}(\ln x) =$ **SCORE: ___ / 3 POINTS**

[a] $\dfrac{2(x^2+1)}{x}$ [b] $\dfrac{2x}{1+x^2}$ [c] $2x^{-1}-2x$ [d] $\dfrac{2x}{x^2-1}$ [e] none of the above

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Simplify $2\sinh x \cosh x$ using the exponential definitions of $\sinh x$ and $\cosh x$. **SCORE: ___ / 3 POINTS**
Write your final answer in terms of hyperbolic functions.

$\dfrac{2\left(\frac{e^x-e^{-x}}{2}\right)\left(\frac{e^x+e^{-x}}{2}\right)}{}$ (1)

$= \dfrac{e^{2x}-e^{-2x}}{2}$ (1)

$= \sinh 2x$ (1)

If $\sinh x = -2$, find $\operatorname{sech} x$ and $\coth x$. **SCORE: ___ / 4 POINTS**

$\cosh^2 x - \sinh^2 x = 1$

$\cosh^2 x = 1 + \sinh^2 x = 5$ ← (1) POINT FOR SHOWING USAGE OF IDENTITY

$\cosh x = \sqrt{5}$ (1)

$\operatorname{sech} x = \dfrac{1}{\cosh x} = \dfrac{1}{\sqrt{5}}$ (1)

$\coth x = \dfrac{\cosh x}{\sinh x} = -\dfrac{\sqrt{5}}{2}$ (1)

Show that $\lim_{x\to\infty} \coth x = 1$. **DO NOT use the value of** $\lim_{x\to\infty} \tanh x$. **SCORE: ___ / 4 POINTS**

$\lim_{x\to\infty} \dfrac{e^x+e^{-x}}{e^x-e^{-x}}$ OR $\lim_{x\to\infty} \dfrac{e^{2x}+1}{e^{2x}-1} = \lim_{x\to\infty} \dfrac{2e^{2x}}{2e^{2x}} 1 = 1$ (4)

$\frac{\infty}{\infty}$ (L'HÔPITAL'S RULE)

OR

OR $\lim_{x\to\infty} \dfrac{1+\frac{1}{e^{2x}}}{1-\frac{1}{e^{2x}}} = \dfrac{1+0}{1-0} = 1$ (3) (1)

Prove the formula for the derivative of $\coth x$.

SCORE: ___ / 4 POINTS

You may use the derivatives of $\sinh x$ and $\cosh x$ without proving them.
DO NOT use the derivative of any other hyperbolic function.

$$\frac{d}{dx}\frac{\cosh x}{\sinh x} = \frac{(\sinh x)\sinh x - \cosh x(\cosh x)}{\sinh^2 x}$$

$$= \underbrace{\frac{\sinh^2 x - \cosh^2 x}{\sinh^2 x}}_{(2)}$$

$$= \underbrace{\frac{-1}{\sinh^2 x}}_{(1)} = \underbrace{-\operatorname{csch}^2 x}_{(1)}$$

Derive the formula $\tanh^{-1} x = \frac{1}{2}\ln\frac{1+x}{1-x}$.

SCORE: ___ / 4 POINTS

NOTE: "DERIVE" means "show how this formula was found", NOT "take the derivative of".

LET $y = \tanh^{-1} x$

so $\tanh y = x$

so $\underbrace{\frac{e^{2y}-1}{e^{2y}+1} = x}_{(1)}$

$\underbrace{e^{2y} - 1 = xe^{2y} + x}_{(1)}$

$e^{2y} - xe^{2y} = 1 + x$

$\underbrace{(1-x)e^{2y} = 1+x}_{(1)}$

$e^{2y} = \frac{1+x}{1-x}$ ← (1) IF YOU HAVE EITHER OR BOTH OF THESE

$2y = \ln\frac{1+x}{1-x}$

$y = \frac{1}{2}\ln\frac{1+x}{1-x} = \tanh^{-1} x$

Find the derivatives of the following functions. **Simplify your final answers.**

SCORE: ___ / 8 POINTS

[a] $f(x) = \tanh^{-1}(\operatorname{sech} x)$

$$f'(x) = \underbrace{\frac{1}{1-\operatorname{sech}^2 x}}_{(1)}\underbrace{(-\operatorname{sech} x\tanh x)}_{(1)}$$

$$= \underbrace{\frac{1}{\tanh^2 x}}_{(1)}(-\operatorname{sech} x\tanh x)$$

$$= \underbrace{\frac{-\operatorname{sech} x}{\tanh x}}_{(\frac{1}{2})}\frac{\cosh x}{\cosh x}$$

$$= \underbrace{\frac{-1}{\sinh x} = -\operatorname{csch} x}_{(\frac{1}{2})}$$

[b] $f(x) = (\sinh^{-1} x)(\cosh^{-1} x)$

$$f'(x) = \underbrace{\frac{1}{\sqrt{x^2+1}}\cosh^{-1} x}_{(2)} + \underbrace{\frac{1}{\sqrt{x^2-1}}\sinh^{-1} x}_{(2)}$$

IT IS OK BUT HIGHLY UNDESIRABLE IF YOU WROTE
$\ln(x+\sqrt{x^2-1})$ FOR $\cosh^{-1} x$
$\ln(x+\sqrt{x^2+1})$ FOR $\sinh^{-1} x$