Math 1B (9:30am – 10:20am)
Quiz 1 Version C
Fri Apr 9, 2010

SCORE: ___ / 30 POINTS

What month is your birthday ? ___ ___
What are the first 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___
What are the last 2 digits of your social security number ? ___ ___
[IF YOU DO NOT HAVE A SOCIAL SECURITY NUMBER, USE YOUR STUDENT ID NUMBER]

NO CALCULATORS ALLOWED
YOU MUST SHOW PROPER WORK (EXCEPT MULTIPLE CHOICE)

[MULTIPLE CHOICE] $\operatorname{csch}(\ln x) =$ **SCORE: ___ / 3 POINTS**

[a] $\dfrac{2(x^2+1)}{x}$ [b] $\dfrac{2x}{1+x^2}$ [c] $2x^{-1}-2x$ [d] $\dfrac{2x}{x^2-1}$ [e] none of the above

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Simplify ½ sech x csch x using the exponential definitions of sech x and csch x. **SCORE: ___ / 3 POINTS**
Write your final answer in terms of hyperbolic functions.

$\frac{1}{2}\,\frac{2}{e^x+e^{-x}}\,\frac{2}{e^x-e^{-x}}$ (1)

$= \dfrac{2}{e^{2x}-e^{-2x}}$ (1)

$= \operatorname{csch} 2x$ (1)

If $\cosh x = 3$ and $\sinh x < 0$, find csch x and coth x. **SCORE: ___ / 4 POINTS**

$\cosh^2 x - \sinh^2 x = 1$
$\cosh^2 x - 1 = \sinh^2 x$
$8 = \sinh^2 x$
$\sinh x = -2\sqrt{2}$ (1)

$\operatorname{csch} x = \dfrac{1}{\sinh x} = -\dfrac{1}{2\sqrt{2}}$ (1)

$\coth x = \dfrac{\cosh x}{\sinh x} = -\dfrac{3}{2\sqrt{2}}$ (1)

(1) IF YOU SHOWED USAGE OF IDENTITY

Show that $\lim_{x\to\infty} \tanh x = 1$. **SCORE: ___ / 4 POINTS**

$\lim_{x\to\infty} \dfrac{e^{2x}-1}{e^{2x}+1} \overset{\frac{\infty}{\infty}}{=} \lim_{x\to\infty} \dfrac{2e^{2x}}{2e^{2x}}\,1 = 1$ (4)

L'HÔPITAL'S RULE

OR

OR

$= \lim_{x\to\infty} \dfrac{1-\frac{1}{e^{2x}}}{1+\frac{1}{e^{2x}}} = \dfrac{1-0}{1+0} = 1$ (3) (1)

Prove the formula for the derivative of $\tanh x$. **SCORE: ___ / 4 POINTS**

You may use the derivatives of $\sinh x$ and $\cosh x$ without proving them.
You may NOT use the derivative of any other hyperbolic function.

$$\frac{d}{dx}\frac{\sinh x}{\cosh x} = \frac{(\cosh x)\cosh x - \sinh x(\sinh x)}{\cosh^2 x}$$

$$= \underbrace{\frac{\cosh^2 x - \sinh^2 x}{\cosh^2 x}}_{(2)}$$

$$= \underbrace{\frac{1}{\cosh^2 x}}_{(1)} = \underbrace{\operatorname{sech}^2 x}_{(1)}$$

Derive the formula $\coth^{-1} x = \frac{1}{2}\ln\frac{x+1}{x-1}$. **SCORE: ___ / 4 POINTS**

NOTE: "DERIVE" means "show how this formula was found", NOT "take the derivative of".

LET $y = \coth^{-1} x$

SO $\coth y = x$

$$\underbrace{\frac{e^{2y}+1}{e^{2y}-1} = x}_{(1)}$$

$$\underbrace{e^{2y}+1 = xe^{2y} - x}_{(1)}$$

$$1 + x = xe^{2y} - e^{2y}$$

$$\underbrace{1 + x = (x-1)e^{2y}}_{(1)}$$

$$\frac{1+x}{x-1} = e^{2y} \quad \leftarrow (1)$$

IF YOU HAVE ONE OR BOTH OF THESE

$$2y = \ln\frac{x+1}{x-1}$$

$$y = \frac{1}{2}\ln\frac{x+1}{x-1} = \coth^{-1} x$$

Find the derivatives of the following functions. **Simplify your final answers.** **SCORE: ___ / 8 POINTS**

[a] $f(x) = \cosh^{-1}\sqrt{x^2+1}$ (assume $x \geq 0$)

$$f'(x) = \underbrace{\frac{1}{\sqrt{x^2+1}-1}\,\frac{1}{2\sqrt{x^2+1}}\,2x}_{(1)\qquad\qquad(1)}$$

$$= \frac{1}{\sqrt{x^2}}\,\frac{x}{\sqrt{x^2+1}}$$

$$= \underbrace{\frac{1}{x}\,\frac{x}{\sqrt{x^2+1}}}_{(1)}$$

$$= \underbrace{\frac{1}{\sqrt{x^2+1}}}_{(1)}$$

[b] $f(x) = (\tanh^{-1} x)(\sinh^{-1} x)$

$$f'(x) = \underbrace{\frac{1}{1-x^2}\sinh^{-1} x}_{(2)} + \underbrace{\frac{1}{\sqrt{x^2+1}}\tanh^{-1} x}_{(2)}$$

IT IS OK BUT HIGHLY UNDESIRABLE IF YOU WROTE

$\ln(x + \sqrt{x^2+1})$ FOR $\sinh^{-1} x$

$\frac{1}{2}\ln\frac{1+x}{1-x}$ FOR $\tanh^{-1} x$