Math 1B (9:30am – 10:20am)
Quiz 1 Version D
Fri Apr 9, 2010

SCORE: ___ / 30 POINTS

What month is your birthday ? ___ ___
What are the first 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___
What are the last 2 digits of your social security number ? ___ ___
[IF YOU DO NOT HAVE A SOCIAL SECURITY NUMBER, USE YOUR STUDENT ID NUMBER]

NO CALCULATORS ALLOWED
YOU MUST SHOW PROPER WORK (EXCEPT MULTIPLE CHOICE)

If $\cosh x = 3$ and $\sinh x < 0$, find $\operatorname{csch} x$ and $\coth x$.

SCORE: ___ / 4 POINTS

$\cosh^2 x - \sinh^2 x = 1$ ← (1) IF YOU SHOWED USAGE OF IDENTITY

$\sinh^2 x = \cosh^2 x - 1 = 8$

$\sinh x = -2\sqrt{2}$ (1)

$\operatorname{csch} x = \frac{1}{\sinh x} = -\frac{1}{2\sqrt{2}}$ (1)

$\coth x = \frac{\cosh x}{\sinh x} = -\frac{3}{2\sqrt{2}}$ (1)

Find the derivatives of the following functions. **Simplify your final answers.**

SCORE: ___ / 8 POINTS

[a] $f(x) = \cosh^{-1}\sqrt{x^2+1}$ (assume $x \geq 0$)

$f'(x) = \frac{1}{\sqrt{x^2+1-1}} \frac{1}{2\sqrt{x^2+1}} 2x$ (1) (1)

$= \frac{1}{\sqrt{x^2}} \frac{x}{\sqrt{x^2+1}}$

$= \frac{1}{x} \frac{x}{\sqrt{x^2+1}}$ (1)

$= \frac{1}{\sqrt{x^2+1}}$ (1)

[b] $f(x) = (\tanh^{-1} x)(\sinh^{-1} x)$

$f'(x) = \frac{1}{1-x^2} \sinh^{-1} x$ (2)

$+ \frac{1}{\sqrt{x^2+1}} \tanh^{-1} x$ (2)

IT IS OK BUT HIGHLY UNDESIRABLE IF YOU WROTE $\ln(x+\sqrt{x^2+1})$ FOR $\sinh^{-1} x$

$\frac{1}{2}\ln\frac{1+x}{1-x}$ FOR $\tanh^{-1} x$

Derive the formula $\coth^{-1} x = \frac{1}{2}\ln\frac{x+1}{x-1}$.

SCORE: ___ / 4 POINTS

NOTE: "DERIVE" means "show how this formula was found", NOT "take the derivative of".

LET $y = \coth^{-1} x$

SO $\coth y = x$

$\frac{e^{2y}+1}{e^{2y}-1} = x$ (1)

$e^{2y}+1 = xe^{2y} - x$ (1)

$x+1 = xe^{2y} - e^{2y}$

$x+1 = (x-1)e^{2y}$ (1)

$\frac{x+1}{x-1} = e^{2y}$ ← (1) IF YOU HAVE ONE OR BOTH OF THESE

$2y = \ln\frac{x+1}{x-1}$

$y = \frac{1}{2}\ln\frac{x+1}{x-1}$

Prove the formula for the derivative of $\tanh x$. **SCORE: ___ / 4 POINTS**

You may use the derivatives of $\sinh x$ and $\cosh x$ without proving them.
You may NOT use the derivative of any other hyperbolic function.

$$\frac{d}{dx}\frac{\sinh x}{\cosh x} = \frac{(\cosh x)\cosh x - \sinh x(\sinh x)}{\cosh^2 x}$$

$$= \frac{\cosh^2 x - \sinh^2 x}{\cosh^2 x} \quad (2)$$

$$= \frac{1}{\cosh^2 x} \quad (1)$$

$$= \operatorname{sech}^2 x \quad (1)$$

Show that $\lim_{x\to\infty} \tanh x = 1$. **SCORE: ___ / 4 POINTS**

$$\lim_{x\to\infty} \frac{e^{2x}-1}{e^{2x}+1} = \lim_{x\to\infty} \frac{2e^{2x}}{2e^{2x}} 1 = 1$$

$\frac{\infty}{\infty}$ (L'HÔPITAL'S RULE) (4)

OR OR

$$= \lim_{x\to\infty} \frac{1-\frac{1}{e^{2x}}}{1+\frac{1}{e^{2x}}} = \frac{1-0}{1+0} = 1$$

(3) (1)

Simplify ½ $\operatorname{sech} x \operatorname{csch} x$ using the exponential definitions of $\operatorname{sech} x$ and $\operatorname{csch} x$. **SCORE: ___ / 3 POINTS**
Write your final answer in terms of hyperbolic functions.

$$\frac{1}{2}\,\frac{2}{e^x+e^{-x}}\,\frac{2}{e^x-e^{-x}} \quad (1)$$

$$= \frac{2}{e^{2x}-e^{-2x}} \quad (1)$$

$$= \operatorname{csch} 2x \quad (1)$$

[MULTIPLE CHOICE] $\operatorname{csch}(\ln x) =$ **SCORE: ___ / 3 POINTS**

[a] $\frac{2(x^2+1)}{x}$ [b] $\frac{2x}{1+x^2}$ [c] $2x^{-1}-2x$ [d] $\frac{2x}{x^2-1}$ [e] none of the above