Math 1B (7:30am – 8:20am)
Quiz 1 Version B
Fri Apr 9, 2010

SCORE: ___ / 30 POINTS

What month is your birthday ?
What are the first 2 digits of your address ?
What are the last 2 digits of your zip code ?
What are the last 2 digits of your social security n
[IF YOU DO NOT HAVE A SOCIAL SECUR
USE YOUR STUDENT ID NUMBER]

NO CALCULATORS ALLOWED
YOU MUST SHOW PROPER WORK (EXCEPT MULTIPLE

Find the derivatives of the following functions. Simplify your final answers.

SCORE: 6 / 8 POINTS

[a] $f(x) = (\sinh^{-1} x)(\cosh^{-1} x)$

$f'(x) = \frac{1}{\sqrt{1+x^2}} \cdot \cosh^{-1} x + \sinh^{-1} x \cdot \frac{1}{\sqrt{x^2-1}}$ (2) (2)

$= \frac{(\sqrt{x^2-1})\cosh^{-1} x + (\sqrt{1+x^2})\sinh^{-1} x}{(\sqrt{1+x^2})(\sqrt{x^2-1})}$

[b] $f(x) = \tanh^{-1}(\operatorname{sech} x)$

$f'(x) = \frac{1}{1-\operatorname{sech}^2 x}(-\operatorname{sech} x \tanh x)$ (1) (1)

$f'(x) = -\frac{\operatorname{sech} x \tanh x}{1-\operatorname{sech}^2 x}$

Show that $\lim_{x\to\infty} \coth x = 1$. DO NOT use the value of $\lim_{x\to\infty} \tanh x$.

SCORE: 3½ / 4 POINTS

$\lim_{x\to\infty} \frac{\cosh x}{\sinh x}$

$\lim_{x\to\infty} \frac{e^{2x}+1}{e^{2x}-1}$

L'H $= \frac{2e^{2x}}{2e^{2x}} = 1$ //

(3½) FORGOT $\lim_{x\to\infty}$

Derive the formula $\tanh^{-1} x = \frac{1}{2}\ln\frac{1+x}{1-x}$.

SCORE: 4 / 4 POINTS

NOTE: "DERIVE" means "show how this formula was found", NOT "take the derivative of".

$\tanh^{-1} x = y$

$\tanh y = x$

$\frac{\sinh y}{\cosh y} = x$

$\frac{e^{2y}-1}{e^{2y}+1} = x$ (1)

$e^{2y}-1 = xe^{2y}+x$ (1)

$e^{2y}-xe^{2y} = x+1$

$e^{2y}(1-x) = x+1$ (1)

$e^{2y} = \frac{x+1}{1-x}$ (1)

$2y = \ln\left(\frac{1+x}{1-x}\right)$

$y = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$ (2)

NEED TO DEFEND THIS

If $\sinh x = -2$, find $\operatorname{sech} x$ and $\coth x$.

SCORE: ___ / 4 POINTS

$\sinh x = -2$

$\sinh^{-1}(-2) = x$

$\ln\left(x + \sqrt{x^2+1}\right) = \sinh^{-1} x$

$\ln\left(-2 + \sqrt{(-2)^2+1}\right) = x$

$\ln\left(-2+\sqrt{5}\right) = x$

$\operatorname{sech} x = \frac{1}{\cosh x}$

$\operatorname{sech} x = \frac{2}{e^x + e^{-x}}$

$= \frac{2}{e^{\ln(-2+\sqrt{5})} + e^{-\ln(-2+\sqrt{5})}}$

$= \frac{2}{(-2+\sqrt{5}) + \frac{1}{(-2+\sqrt{5})}}$

$= \frac{2}{(-2+\sqrt{5})(-2+\sqrt{5}) + 1}$

$= \frac{2(-2+\sqrt{5})}{-2\sqrt{5}+10}$

$= \frac{2(-2+\sqrt{5})}{-4+2\sqrt{5}+5+1}$

$= \frac{2(-2+\sqrt{5})}{2(-\sqrt{5}+5)}$

$\coth x = \frac{\cosh x}{\sinh x}$

$= \frac{e^x + e^{-x}}{2} \cdot \frac{1}{(-2)}$

$= \frac{e^{\ln(-2+\sqrt{5})} + e^{-\ln(-2+\sqrt{5})}}{2} \cdot \frac{1}{(-2)}$

$= \frac{-2+\sqrt{5} + \frac{1}{-2+\sqrt{5}}}{2} \cdot \frac{1}{-2}$

$= \frac{(-2+\sqrt{5})(-2+\sqrt{5}) + 1}{-4(-2+\sqrt{5})}$

$= \frac{-2\sqrt{5}+10}{-4(-2+\sqrt{5})}$

Simplify $2\sinh x \cosh x$ using the exponential definitions of $\sinh x$ and $\cosh x$.
Write your final answer in terms of hyperbolic functions.

SCORE: 1 / 3 POINTS

$2\left(\frac{e^x - e^{-x}}{2}\right)\left(\frac{e^x + e^{-x}}{2}\right)$ (1)

$\frac{2e^x - 2e^{-x}}{2} \cdot \frac{2(e^x - e^{-x})}{2} \cdot \frac{2}{2}$

$= 2\left(\frac{e^x - e^{-x}}{2}\right)$

$= 2\sinh x$

Prove the formula for the derivative of $\coth x$. $= \frac{\cosh x}{\sinh x} \rightarrow -\operatorname{cosec}^2 x$

SCORE: 4 / 4 POINTS

You may use the derivatives of $\sinh x$ and $\cosh x$ without proving them.
DO NOT use the derivative of any other hyperbolic function.

$\cosh^2 x - \sinh^2 x = 1$

$\coth x = y$

$\frac{\cosh x}{\sinh x} = y$

$\frac{\sinh x \sinh x - \cosh x \cosh x}{\sinh^2 x} = \frac{dy}{dx}$

$\frac{\sinh^2 x - \cosh^2 x}{\sinh^2 x} = \frac{dy}{dx}$ (2)

$\frac{-(\cosh^2 x - \sinh^2 x)}{\sinh^2 x} = \frac{dy}{dx}$

$\frac{-1}{\sinh^2 x} = \frac{dy}{dx}$ (1)

$-\operatorname{cosech}^2 x = \frac{dy}{dx}$ (1)

[MULTIPLE CHOICE] $\operatorname{sech}(\ln x) =$

$\frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}} = \frac{2}{e^{\ln x} + e^{-\ln x}} = \frac{2}{x + \frac{1}{x}} = \frac{2x}{x^2+1}$

SCORE: 3 / 3 POINTS

[a] $\frac{2(x^2+1)}{x}$ [b] $\frac{2x}{x^2-1}$ [c] $2x^{-1} - 2x$ [d] $\frac{2x}{1+x^2}$ [e] none of the above