Math 1B (7:30am – 8:20am)
Quiz 1 Version A
Fri Apr 16, 2010

SCORE: ___ / 30 POINTS

What month is your birthday ? ___ ___
What are the first 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___
What are the last 2 digits of your social security number ? ___ ___
[IF YOU DO NOT HAVE A SOCIAL SECURITY NUMBER, USE YOUR STUDENT ID NUMBER]

NO CALCULATORS ALLOWED
YOU MUST SHOW PROPER CALCULUS LEVEL WORK

State the definition of "definite integral". **SCORE: ___ / 2 POINTS**

THE DEFINITE INTEGRAL OF f ON $[a,b]$ IS
$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$ WHERE $\Delta x = \frac{b-a}{n}$ AND $a+(i-1)\Delta x \le x_i^* \le a+i\Delta x$
IF THE LIMIT EXISTS

State the Fundamental Theorem of Calculus Part 1. **SCORE: ___ / 2 POINTS**

IF f IS CONTINUOUS ON $[a,b]$
THEN $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$ FOR ALL $x \in (a,b)$

Use the definition of the definite integral, and right endpoints, to evaluate $\int_{-3}^{1} (1+4x)\,dx$. **SCORE: ___ / 6 POINTS**

$\lim_{n\to\infty} \sum_{i=1}^{n} f(-3+\frac{4i}{n})\frac{4}{n}$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(1+4\left(-3+\frac{4i}{n}\right)\right)\frac{4}{n}$ (1) (1) (1)

$= \lim_{n\to\infty} \frac{4}{n} \sum_{i=1}^{n} \left(-11+\frac{16i}{n}\right)$

$= \lim_{n\to\infty} \frac{4}{n} \left(\sum_{i=1}^{n} -11 + \frac{16}{n}\sum_{i=1}^{n} i \right)$ (½) (½)

$= \lim_{n\to\infty} \frac{4}{n} \left(-11n + \frac{16}{n}\cdot\frac{n(n+1)}{2} \right)$ (½) (½)

$= \lim_{n\to\infty} 4\left(-11+\frac{8(n+1)}{n}\right)$ (½)

$= 4(-11+8)$

$= -12$ (½)

Suppose $\int_2^3 \arctan x\,dx = 1.2$, $\int_3^6 \arctan x\,dx = 4.0$ and $\int_4^6 \arctan x\,dx = 2.7$. **SCORE: ___ / 8 POINTS**

[a] Find $\int_6^4 (5-3\arctan x)\,dx$.

$= \int_6^4 5\,dx - 3\int_6^4 \arctan x\,dx$ (½) (1)

$= 5(4-6) - 3\left(-\int_4^6 \arctan x\,dx\right)$ (½)

$= -10 - 3(-2.7)$ (1)

$= -1.9$ (1)

[b] Find $\int_2^4 \arctan x\,dx$. **HINT: Find** $\int_2^6 \arctan x\,dx$.

$= \int_2^3 \arctan x\,dx + \int_3^6 \arctan x\,dx$ (1)
$+ \int_6^4 \arctan x\,dx$ (½)

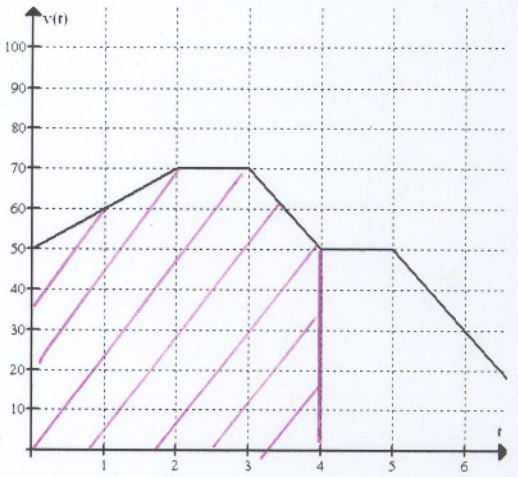
$= 1.2 + 4.0 - \int_4^6 \arctan x\,dx$ (½)

$= 5.2 - 2.7 = 2.5$ (1) (1)

FOR THE FOLLOWING QUESTIONS, YOU MUST SHOW HOW YOU FOUND YOUR ANSWERS. HOWEVER, YOU DO NOT HAVE TO USE FORMAL CALCULUS NOTATION.

The velocity of a car as a function of time $v(t)$ is shown in the graph below. Find the total distance travelled by the car from $t = 0$ to $t = 4$.

SCORE: ___ / 3 POINTS



$\frac{1}{2}(70+50)2 + 70(1) + \frac{1}{2}(70+50)1$ (2)

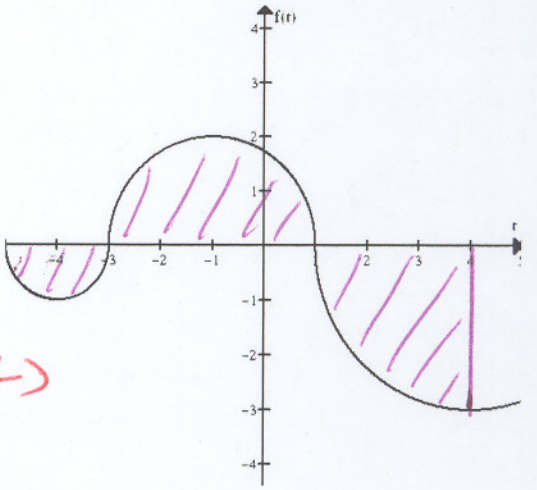
$= 120 + 70 + 60$

$= 250$ (1)

CAN USE ANY CALCULATION WHICH CORRESPONDS TO THE AREA SHADED PURPLE

The graph of $f(t)$ shown below consists of 3 semicircles of radii 1, 2 and 3. Let $g(x) = \int_{-5}^{x} f(t)\,dt$.

SCORE: ___ / 6 POINTS



[a] Find $g(4)$.

(1) $-\frac{1}{2}\pi(1)^2 + \frac{1}{2}\pi(2)^2 - \frac{1}{4}\pi(3)^2$ (1) (1)

$= -\frac{\pi}{2} + 2\pi - \frac{9}{4}\pi$

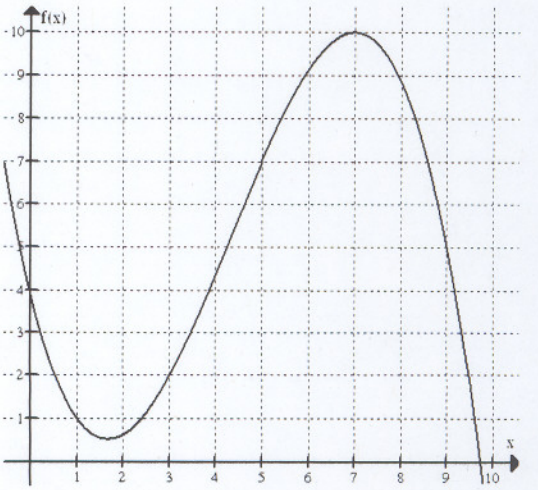
$= \frac{-3\pi}{4}$ (1) ← NO PARTIAL CREDIT IF THIS IS WRONG DUE TO SIGN (+/-) ERROR ABOVE

[b] Find $g'(-1)$.

$g'(-1) = f(-1) = 2$ (1) (1)

The graph of $f(x)$ is shown below. Estimate $\int_{4}^{10} f(x)\,dx$ using 3 subintervals with midpoints.

SCORE: ___ / 4 POINTS



$\Delta x = \frac{10-4}{3} = 2$ ($\frac{1}{2}$)

INTERVALS = [4,6], [6,8], [8,10]

MIDPOINTS = 5, 7, 9

$f(5)\Delta x + f(7)\Delta x + f(9)\Delta x$

(1) $7(2) + 10(2) + 5(2)$ (1) (1)

$= 44$ ($\frac{1}{2}$)