

NO CALCULATORS ALLOWED

YOU MUST SHOW PROPER CALCULUS LEVEL WORK

State the definition of "definite integral".

SCORE: ___ / 2 POINTS

SEE 7:30 VERSION A KEY

State the Fundamental Theorem of Calculus Part 1.

SCORE: ___ / 2 POINTS

SEE 7:30 VERSION A KEY

Use the definition of the definite integral, and right endpoints, to evaluate $\int_{-1}^3 (1+4x) dx$.

SCORE: ___ / 6 POINTS

$$\begin{aligned} & \lim_{n \rightarrow \infty} \sum_{i=1}^n f\left(-1 + \frac{4i}{n}\right) \frac{4}{n} \\ &= \lim_{n \rightarrow \infty} \sum_{i=1}^n \left(1 + 4\left(-1 + \frac{4i}{n}\right)\right) \frac{4}{n} \\ &= \lim_{n \rightarrow \infty} \frac{4}{n} \sum_{i=1}^n \left(-3 + \frac{16i}{n}\right) \\ &= \lim_{n \rightarrow \infty} \frac{4}{n} \left(\sum_{i=1}^n -3 + \frac{16}{n} \sum_{i=1}^n i \right) \\ &= \lim_{n \rightarrow \infty} \frac{4}{n} \left(-3n + \frac{16}{n} \frac{n(n+1)}{2} \right) \\ &= \lim_{n \rightarrow \infty} 4 \left(-3 + \frac{8(n+1)}{n} \right) \\ &= 4(-3 + 8) \\ &= 20 \end{aligned}$$

Suppose $\int_1^2 \arctan x dx = 1.0$, $\int_2^5 \arctan x dx = 3.8$ and $\int_3^5 \arctan x dx = 2.6$.

SCORE: ___ / 8 POINTS

[a] Find $\int_5^3 (6 - 4 \arctan x) dx$.

$$\begin{aligned} &= \int_5^3 6 dx - 4 \int_5^3 \arctan x dx \\ &= 6(3-5) + 4 \int_3^5 \arctan x dx \\ &= -12 + 4(2.6) \\ &= -1.6 \end{aligned}$$

[b] Find $\int_1^3 \arctan x dx$. **HINT:** Find $\int_1^5 \arctan x dx$.

$$\begin{aligned} &= \int_1^2 \arctan x dx + \int_2^5 \arctan x dx \\ &= 1.0 + 3.8 - \int_3^5 \arctan x dx \\ &= 4.8 - 2.6 = 2.2 \end{aligned}$$

FOR THE FOLLOWING QUESTIONS, YOU MUST SHOW HOW YOU FOUND YOUR ANSWERS. HOWEVER, YOU DO NOT HAVE TO USE FORMAL CALCULUS NOTATION.

The velocity of a car as a function of time $v(t)$ is shown in the graph below.

SCORE: ___ / 3 POINTS

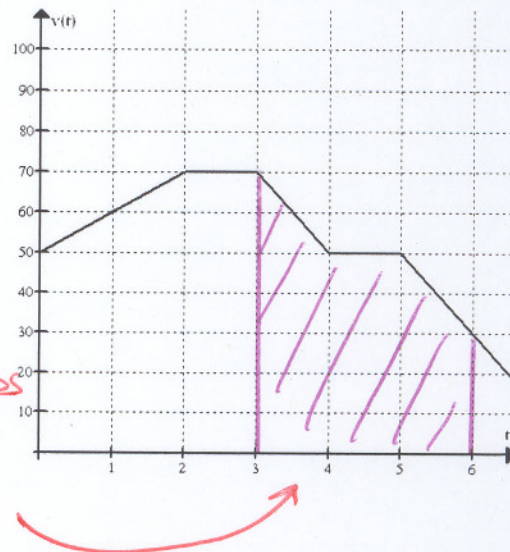
Find the total distance travelled by the car from $t = 3$ to $t = 6$.

$$\frac{1}{2}(70+50)1 + 50(1) + \frac{1}{2}(50+30)1$$

$$= 60 + 50 + 40$$

$$= 150$$

CAN USE ANY CALCULATION WHICH CORRESPONDS TO PURPLE SHADED AREA



The graph of $f(t)$ shown below consists of 3 semicircles of radii 3, 1 and 2. Let $g(x) = \int_{-4}^x f(t) dt$.

SCORE: ___ / 6 POINTS

[a] Find $g(3)$.

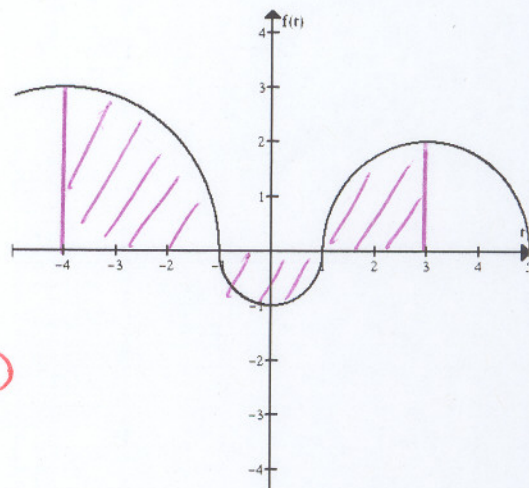
$$\frac{1}{4}\pi(3)^2 - \frac{1}{2}\pi(1)^2 + \frac{1}{4}\pi(2)^2$$

$$= \frac{9\pi}{4} - \frac{\pi}{2} + \pi$$

$$= \frac{11\pi}{4}$$

[b] Find $g'(3)$.

$$g'(3) = f(3) = 2$$



The graph of $f(x)$ is shown below. Estimate $\int_1^7 f(x) dx$ using 3 subintervals with midpoints.

SCORE: ___ / 4 POINTS

$$\Delta x = \frac{7-1}{3} = 2$$

$$\text{INTERVALS} = [1, 3], [3, 5], [5, 7]$$

$$\text{MIDPOINTS} = 2, 4, 6$$

$$f(2)\Delta x + f(4)\Delta x + f(6)\Delta x$$

$$10(2) + 7(2) + 2(2)$$

$$= 38$$

