

SCORE: \_\_\_ / 30 POINTS

What month is your birthday ?

What are the first 2 digits of your address ?

What are the last 2 digits of your zip code ?

What are the last 2 digits of your social security number ?

[IF YOU DO NOT HAVE A SOCIAL SECURITY NUMBER,  
USE YOUR STUDENT ID NUMBER]

## NO CALCULATORS ALLOWED

## YOU MUST SHOW PROPER CALCULUS LEVEL WORK

State the definition of "definite integral".

SCORE: \_\_\_ / 2 POINTS

SEE 7:30 VERSION A KEY

State the Fundamental Theorem of Calculus Part 1.

SCORE: \_\_\_ / 2 POINTS

SEE 7:30 VERSION A KEY

Use the definition of the definite integral, and right endpoints, to evaluate  $\int_{-1}^3 (2+4x) dx$ .

SCORE: \_\_\_ / 6 POINTS

$$\begin{aligned}
 & \lim_{n \rightarrow \infty} \sum_{i=1}^n f(-1 + \frac{4i}{n}) \frac{4}{n} \\
 &= \lim_{n \rightarrow \infty} \sum_{i=1}^n (2 + 4(-1 + \frac{4i}{n})) \frac{4}{n} \\
 &= \lim_{n \rightarrow \infty} \sum_{i=1}^n (-2 + \frac{16i}{n}) \frac{4}{n} \\
 &= \lim_{n \rightarrow \infty} \frac{4}{n} \left( \sum_{i=1}^n -2 + \frac{16}{n} \sum_{i=1}^n i \right) \\
 &= \lim_{n \rightarrow \infty} \frac{4}{n} \left( -2n + \frac{16}{n} \cdot \frac{n(n+1)}{2} \right) \\
 &= \lim_{n \rightarrow \infty} 4 \left( -2 + \frac{8(n+1)}{n} \right) \\
 &= 4(-2 + 8) \\
 &= 24
 \end{aligned}$$

Suppose  $\int_1^2 \arctan x dx = 1.0$ ,  $\int_2^5 \arctan x dx = 3.8$  and  $\int_3^5 \arctan x dx = 2.6$ .

SCORE: \_\_\_ / 8 POINTS

[a] Find  $\int_5^3 (7 - 4 \arctan x) dx$ .

$$\begin{aligned}
 &= \int_5^3 7 dx - 4 \int_5^3 \arctan x dx \\
 &= 7(3-5) + 4 \int_3^5 \arctan x dx \\
 &= -14 + 4(2.6) \\
 &= -3.6
 \end{aligned}$$

[b] Find  $\int_1^3 \arctan x dx$ . HINT: Find  $\int_1^5 \arctan x dx$ .

$$\begin{aligned}
 &= \int_1^2 \arctan x dx + \int_2^5 \arctan x dx \\
 &= 1.0 + 3.8 - \int_3^5 \arctan x dx \\
 &= 4.8 - 2.6 = 2.2
 \end{aligned}$$



FOR THE FOLLOWING QUESTIONS, YOU MUST SHOW HOW YOU FOUND YOUR ANSWERS.  
HOWEVER, YOU DO **NOT** HAVE TO USE FORMAL CALCULUS NOTATION.

The velocity of a car as a function of time  $v(t)$  is shown in the graph below.

SCORE: \_\_\_ / 3 POINTS

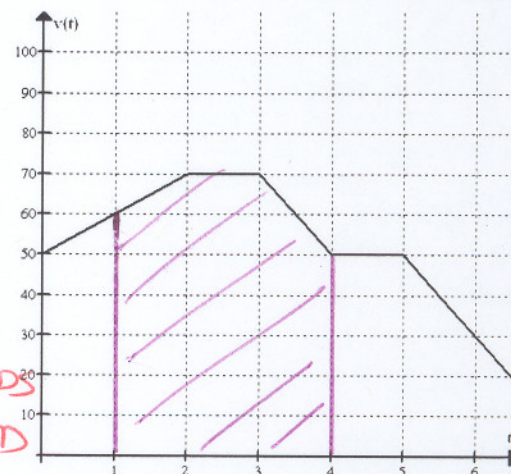
Find the total distance travelled by the car from  $t = 1$  to  $t = 4$ .

$$\frac{1}{2}(60+70)1 + 70(1) + \frac{1}{2}(70+50)1$$

$$= 65 + 70 + 60$$

$$= 195$$

CAN USE ANY  
CALCULATION  
WHICH CORRESPONDS  
TO PURPLE SHADED  
AREA



The graph of  $f(t)$  shown below consists of 3 semicircles of radii 1, 2 and 3. Let  $g(x) = \int_{-4}^x f(t) dt$ .

SCORE: \_\_\_ / 6 POINTS

[a] Find  $g(4)$ .

$$\frac{1}{4}\pi(1)^2 + \frac{1}{2}\pi(2)^2 - \frac{1}{4}\pi(3)^2$$

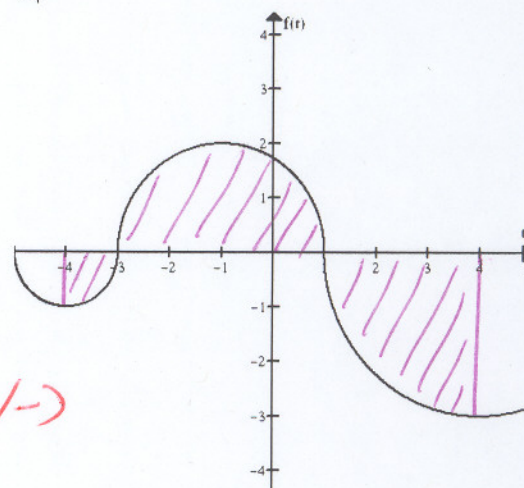
$$= -\frac{\pi}{4} + 2\pi - \frac{9\pi}{4}$$

$$= -\frac{\pi}{2}$$

[b] Find  $g'(4)$ .

$$g'(4) = f(4) = -3$$

NO PARTIAL CREDIT  
IF THIS IS WRONG  
DUE TO SIGN (+/-)  
ERROR ABOVE



The graph of  $f(x)$  is shown below. Estimate  $\int_2^8 f(x) dx$  using 3 subintervals with midpoints.

SCORE: \_\_\_ / 4 POINTS

$$\Delta x = \frac{8-2}{3} = 2$$

$$\text{INTERVALS} = [2, 4], [4, 6], [6, 8]$$

$$\text{MIDPOINTS} = 3, 5, 7$$

$$f(3)\Delta x + f(5)\Delta x + f(7)\Delta x$$

$$= 2(2) + 7(2) + 10(2)$$

$$= 38$$

