

SCORE: \_\_\_ / 30 POINTS

What month is your birthday? \_\_\_\_\_

What are the first 2 digits of your address? \_\_\_\_\_

What are the last 2 digits of your zip code? \_\_\_\_\_

What are the last 2 digits of your social security number? \_\_\_\_\_

[IF YOU DO NOT HAVE A SOCIAL SECURITY NUMBER,  
USE YOUR STUDENT ID NUMBER]

## NO CALCULATORS ALLOWED YOU MUST SHOW PROPER CALCULUS LEVEL WORK

State the definition of "definite integral". [Same question and answer as quiz #2]

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SEE 7:30 VERSION A KEY

State the Net Change Theorem.

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SEE 7:30 VERSION B KEY

If  $c(l)$  is the number of calories you were burning per mile during a marathon after you had run  $l$  miles of it,

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what is the practical meaning of  $\int_2^6 c(l) dl = 500$ ? Give the units for each number in your answer.Your answer should make sense to a 10 year old who has never heard of calculus before.

YOU BURNED 500 CALORIES <sup>RUNNING</sup> BETWEEN THE 2 MILE  
POINT ON A MARATHON AND THE 6 MILE POINT.

The graph of  $f$  is shown below. Let  $g(x) = \int_{-2}^x f(t) dt$ .

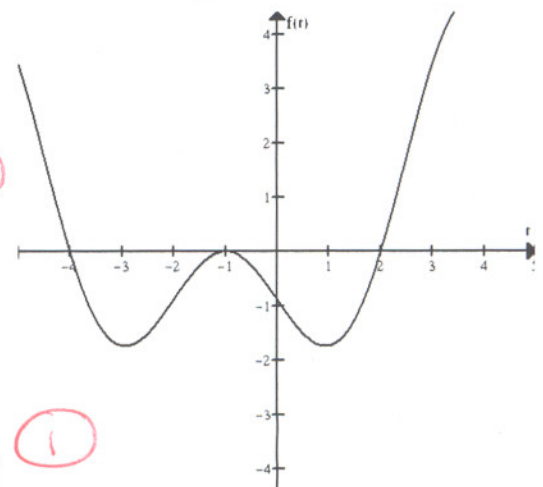
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[a] On what intervals is  $g$  concave up? Explain briefly.

$g' = f$  IS INCREASING ON  $[-3, -1]$  ①  
AND  $[1, \infty)$  ①

[b] At what value(s) of  $x$  does  $g$  have a local minimum (minima)?  
Explain briefly.

$g' = f$  CHANGES FROM NEGATIVE  
TO POSITIVE ①  
AT  $x = 2$  ①



The velocity of an object at time  $t$  seconds is  $v(t) = 6 - 3\sqrt{t}$  feet per second. Find the distance travelled by the object over the interval  $[1, 9]$ . SCORE: \_\_\_ / 6 POINTS

SEE 7:30 VERSION A KEY

Suppose  $f'$  is continuous. If  $f(-1) = 17$  and  $\int_{-1}^2 f'(t) dt = 23$ , find  $f(2)$ .

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$$\int_{-1}^2 f'(t) dt = f(2) - f(-1)$$

$$23 = f(2) - 17$$

$$f(2) = 40 \text{ (1)}$$

① IF YOU HAVE EITHER ONE

Find the derivative of the function  $g(x) = \int_{\sqrt{x}}^{\tan^{-1}x} \ln(1+t^2) dt$ .

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$$g'(x) = \frac{d}{dx} \int_{\sqrt{x}}^{\tan^{-1}x} \ln(1+t^2) dt$$

$$= \frac{d}{dx} \int_{\sqrt{x}}^0 \ln(1+t^2) dt + \frac{d}{dx} \int_0^{\tan^{-1}x} \ln(1+t^2) dt \text{ (1)}$$

$$\text{① } = -\frac{d}{dx} \int_0^{\sqrt{x}} \ln(1+t^2) dt + \frac{d}{dx} \int_0^{\tan^{-1}x} \ln(1+t^2) dt$$

$$= -\frac{d}{d\sqrt{x}} \int_0^{\sqrt{x}} \ln(1+t^2) dt \cdot \frac{d\sqrt{x}}{dx} + \frac{d}{d\tan^{-1}x} \int_0^{\tan^{-1}x} \ln(1+t^2) dt \cdot \frac{d\tan^{-1}x}{dx}$$

$$= -\ln(1+x) \cdot \frac{1}{2\sqrt{x}} + \ln(1+(\tan^{-1}x)^2) \cdot \frac{1}{1+x^2}$$

$$= \frac{-\ln(1+x)}{2\sqrt{x}} + \frac{\ln(1+(\tan^{-1}x)^2)}{1+x^2} \text{ (1)}$$

Evaluate  $\int \sec^3 x \tan x dx$ .

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①  $u = \sec x$

$$\frac{du}{dx} = \sec x \tan x$$

$$du = \sec x \tan x dx$$

①

$$\int \sec^2 x \sec x \tan x dx$$

$$= \int u^2 du \text{ (1)}$$

$$= \frac{1}{3} u^3 + C$$

$$\text{① } = \frac{1}{3} \sec^3 x + C \text{ (1/2)}$$