Math 1B (9:30am – 10:20am)
Quiz 3 Version D
Fri Apr 23, 2010

SCORE: ___ / 30 POINTS

What month is your birthday ? ___ ___
What are the first 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___
What are the last 2 digits of your social security number ? ___ ___
[IF YOU DO NOT HAVE A SOCIAL SECURITY NUMBER, USE YOUR STUDENT ID NUMBER]

NO CALCULATORS ALLOWED
YOU MUST SHOW PROPER CALCULUS LEVEL WORK

State the definition of "definite integral". **[Same question and answer as quiz #2]** **SCORE: ___ / 2 POINTS**

SEE 7:30 VERSION A KEY

State the Fundamental Theorem of Calculus Part 2. **SCORE: ___ / 2 POINTS**

SEE 7:30 VERSION A KEY

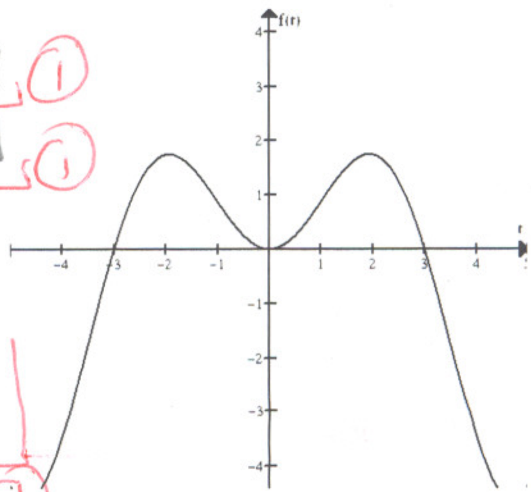
If $c(l)$ is the number of calories you were burning per mile during a marathon after you had run l miles of it, **SCORE: ___ / 2 POINTS**
what is the practical meaning of $\int_{10}^{15} c(l)\,dl = 600$? Give the units for each number in your answer.
Your answer should make sense to a 10 year old who has never heard of calculus before.

YOU BURNED 600 CALORIES RUNNING BETWEEN THE 10 MILE POINT ON A MARATHON AND THE 15 MILE POINT.

The graph of f is shown below. Let $g(x) = \int_{-1}^{x} f(t)\,dt$. **SCORE: ___ / 6 POINTS**

[a] On what intervals is g concave down ? Explain briefly.

$\underbrace{g' = f}_{1}$ IS $\underbrace{\text{DECREASING}}_{1}$ ON $\underbrace{[-2, 0]}$ 1
AND $\underbrace{[2, \infty]}$ 1



[b] At what value(s) of x does g have a local maximum (maxima) ?
Explain briefly.

$g' = f$ $\underbrace{\text{CHANGES FROM POSITIVE TO NEGATIVE}}$ 1
$\underbrace{\text{AT } x = 3}_{1}$

Suppose f' is continuous. If $f(-1) = 7$ and $\int_{-1}^{2} f'(t)\,dt = 11$, find $f(2)$. **SCORE: ___ / 2 POINTS**

$$\int_{-1}^{2} f'(t)\,dt = f(2) - f(-1) \quad ①$$

$$11 = f(2) - 7$$

① IF YOU HAVE EITHER ONE

$$f(2) = 18 \quad ①$$

Find the derivative of the function $g(x) = \int_{\ln x}^{\sqrt{x}} \tan^{-1}(1+t^2)\,dt$. **SCORE: ___ / 5 POINTS**

$$g'(x) = \frac{d}{dx}\int_{\ln x}^{\sqrt{x}} \tan^{-1}(1+t^2)\,dt$$

$$① = \frac{d}{dx}\int_{\ln x}^{0} \tan^{-1}(1+t^2)\,dt + \frac{d}{dx}\int_{0}^{\sqrt{x}} \tan^{-1}(1+t^2)\,dt$$

$$\tfrac{1}{2} = -\frac{d}{dx}\int_{0}^{\ln x} \tan^{-1}(1+t^2)\,dt + \frac{d}{dx}\int_{0}^{\sqrt{x}} \tan^{-1}(1+t^2)\,dt$$

$$= -\frac{d}{d\ln x}\int_{0}^{\ln x} \tan^{-1}(1+t^2)\,dt \cdot \frac{d\ln x}{dx} + \frac{d}{d\sqrt{x}}\int_{0}^{\sqrt{x}} \tan^{-1}(1+t^2)\,dt \cdot \frac{d\sqrt{x}}{dx}$$

$$\tfrac{1}{2} = -\tan^{-1}(1+\ln^2 x)\ \tfrac{1}{2} \cdot \frac{1}{x}\ ① + \tan^{-1}(1+x)\ \tfrac{1}{2} \cdot \frac{1}{2\sqrt{x}}\ ①$$

$$= \frac{\tan^{-1}(1+x)}{2\sqrt{x}} - \frac{\tan^{-1}(1+\ln^2 x)}{x}$$

The velocity of an object at time t seconds is $v(t) = 6 - 3\sqrt{t}$ feet per second. Find the distance travelled by the object over the interval $[1, 9]$. **SCORE: ___ / 6 POINTS**

SEE 7:30 VERSION A KEY

Evaluate $\int \sec^9 x \tan x\,dx$. **SCORE: ___ / 5 POINTS**

① $u = \sec x$

$\frac{du}{dx} = \sec x \tan x$

① $du = \sec x \tan x\,dx$

$$\int \sec^8 x \sec x \tan x\,dx$$

$$= \int u^8\,du \quad ①$$

$$① = \frac{1}{9}u^9 + C$$

$$\tfrac{1}{2} = \frac{1}{9}\sec^9 x + C \quad \tfrac{1}{2}$$