Math 1B (7:30am – 8:20am)
Quiz 4 Version A
Fri May 7, 2010

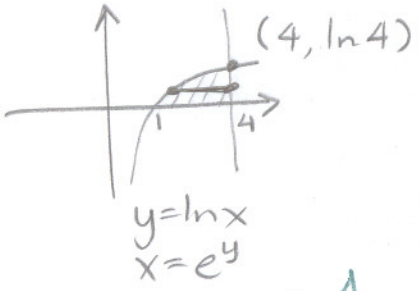
SCORE: ___ / 30 POINTS

What month is your birthday ? ___ ___
What are the first 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___
What are the last 2 digits of your social security number ? ___ ___
[IF YOU DO NOT HAVE A SOCIAL SECURITY NUMBER, USE YOUR STUDENT ID NUMBER]

NO CALCULATORS ALLOWED

Find the area of the region bounded by $y = \ln x$, $x = 4$ and $y = 0$.

SCORE: ___ / 7 POINTS



$$\int_0^{\ln 4} (4 - e^y)\,dy = (4y - e^y)\Big|_0^{\ln 4} = 4\ln 4 - e^{\ln 4} - (0 - 1) = 4\ln 4 - 4 + 1 = 4\ln 4 - 3$$

OR

YOU MAY ONLY EARN POINTS FOR 1 SOLUTION

OR

$$\int_1^4 \ln x\,dx = (x\ln x - x)\Big|_1^4 = 4\ln 4 - 4 - (0 - 1) = 4\ln 4 - 3$$

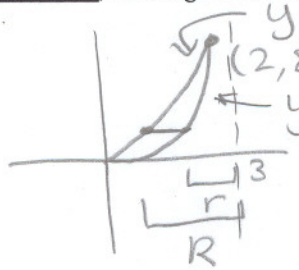
THIS IS AN ANTI-DERIVATIVE YOU HAVEN'T LEARNED IN THIS CLASS

The region bounded by $y = x^3$ and $y = 2x^2$ is revolved around $x = 3$.

SCORE: ___ / 8 POINTS

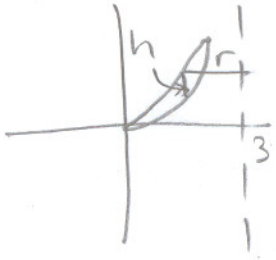
[a] Write, **BUT DO NOT EVALUATE**, an integral for the volume of the solid **USING THE DISK OR WASHER METHOD**.

$x^3 = 2x^2$
$x^3 - 2x^2 = 0$
$x^2(x-2) = 0$
$x = 0, 2$
TEST AT $x = 1$
$1^3 < 2(1)^2$



$$\int_0^8 \pi\left[\left(3 - \sqrt{\tfrac{y}{2}}\right)^2 - \left(3 - \sqrt[3]{y}\right)^2\right] dy$$

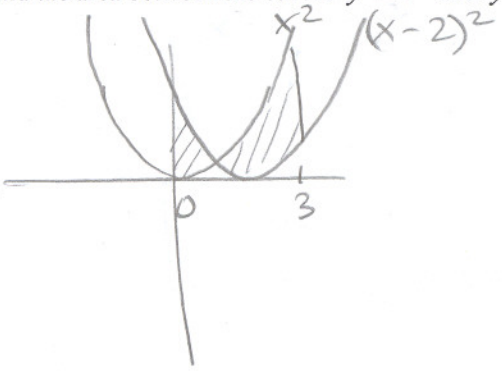
[b] Write, **BUT DO NOT EVALUATE**, an integral for the volume of the solid **USING THE SHELL METHOD**.



$$\int_0^2 2\pi\,(3 - x)(2x^2 - x^3)\,dx$$

Find the area between the curves $y = x^2$ and $y = (x-2)^2$ on the interval $0 \le x \le 3$.

SCORE: ___ / 7 POINTS



$x^2 = (x-2)^2$

$x^2 = x^2 - 4x + 4$

$4x = 4$

$x = 1$

$$\int_0^1 ((x-2)^2 - x^2)\,dx + \int_1^3 (x^2 - (x-2)^2)\,dx$$

$$= \int_0^1 (-4x+4)\,dx + \int_1^3 (4x-4)\,dx$$

$$= (-2x^2+4x)\Big|_0^1 + (2x^2-4x)\Big|_1^3$$

$$= (-2+4) - 0 + (18-12) - (2-4)$$

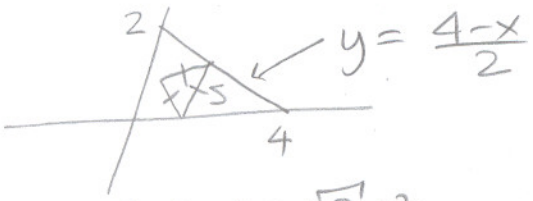
$$= 10$$

The base of a solid is the region in the first quadrant bounded by $x + 2y = 4$ and the $x-$ and $y-$ axes.

SCORE: ___ / 4 POINTS

Cross sections perpendicular to the $x-$ axis are equilateral triangles.

Write, **BUT DO NOT EVALUATE**, an integral for the volume of the solid.



$y = \frac{4-x}{2}$

AREA $= \frac{\sqrt{3}}{4}s^2$

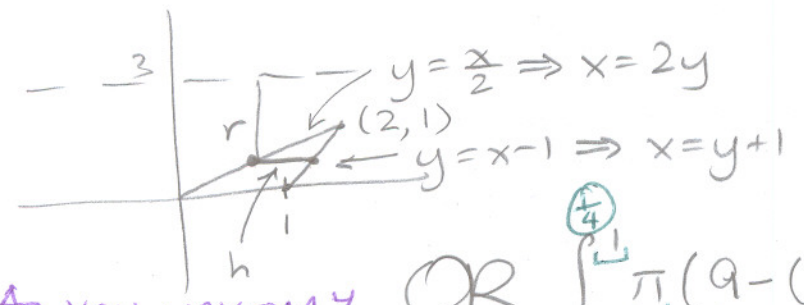
$s = \frac{4-x}{2} - 0 = \frac{4-x}{2}$

$$\int_0^4 \frac{\sqrt{3}}{4}\left(\frac{4-x}{2}\right)^2 dx$$

The region bounded by $y = \frac{x}{2}$, $y = x - 1$ and $y = 0$ is revolved around $y = 3$.

SCORE: ___ / 4 POINTS

Write, **BUT DO NOT EVALUATE**, an integral for the volume of the solid.



$y = \frac{x}{2} \Rightarrow x = 2y$

$y = x - 1 \Rightarrow x = y + 1$

$$\int_0^1 2\pi(3-y)(y+1-2y)\,dy$$

$$= \int_0^1 2\pi(3-y)(1-y)\,dy$$

OR

$$\int_0^1 \pi\left(9 - \left(3 - \frac{x}{2}\right)^2\right)dx + \int_1^2 \pi\left[(3-(x-1))^2 - \left(3 - \frac{x}{2}\right)^2\right]dx$$

★ YOU MAY ONLY EARN POINTS FOR 1 SOLUTION