Math 1B (9:30am – 10:20am)
Quiz 5 Version C
Fri May 14, 2010

SCORE: ___ / 30 POINTS

What month is your birthday ? ___ ___
What are the first 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___
What are the last 2 digits of your social security number ? ___ ___
[IF YOU DO NOT HAVE A SOCIAL SECURITY NUMBER, USE YOUR STUDENT ID NUMBER]

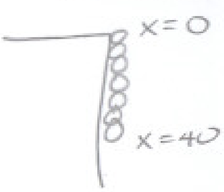
YOU MUST SHOW PROPER CALCULUS LEVEL WORK TO EARN FULL CREDIT.
UNLESS OTHERWISE SPECIFIED, CALCULATORS MAY BE USED ONLY TO CHECK ANSWERS.

BONUS QUESTION: **SCORE: ___ / ___ POINTS**

A chain of constant linear density hangs over the edge of a building. The chain does not reach the ground. Mary Kate and Ashley must pull the chain to the roof of the building, and they want to split the work evenly. Show that the percentage of the chain the first person must pull does not depend on the density or length of the chain, then find that percentage. (Assume British units.)

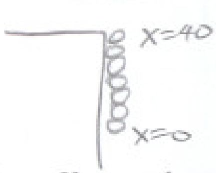
A 40 foot chain weighing 6 pounds per foot hangs over the edge of a 240 foot tall building. **SCORE: ___ / 12 POINTS**

[a] How much work is done in pulling the chain to the top of the building ?
NO CREDIT FOR USING NON-CALCULUS-BASED PHYSICS SOLUTIONS.



$$\int_0^{40} 6x\,dx = 3x^2\Big|_0^{40} = 3(40)^2 = 4800 \text{ ft-lb}$$

OR



$$\int_0^{40} 6(40-x)\,dx = (240x-3x^2)\Big|_0^{40} = 240(40)-3(40)^2$$
$$= 4800 \text{ ft-lb}$$

[b] How much work is done in pulling one quarter of the chain to the top of the building ?
NO CREDIT FOR USING NON-CALCULUS-BASED PHYSICS SOLUTIONS.

$$\int_0^{10} 6x\,dx + \int_{10}^{40} 6(10)\,dx = 3x^2\Big|_0^{10} + 1800 = 3(10)^2 + 1800 = 2100 \text{ ft-lb}$$

OR

CREDIT FOR ONLY 1 OF THESE

$$\int_{30}^{40} 6(40-x)\,dx + \int_0^{30} 6(10)\,dx = (240x-3x^2)\Big|_{30}^{40} + 1800$$

OR

OR THIS

CREDIT FOR ONLY THIS

$$= 240(40-30) - 3(40^2-30^2) + 1800$$
$$= 2100 \text{ ft-lb}$$

OR

$$\int_{30}^{40} 6x\,dx = 3x^2\Big|_{30}^{40} = 3(40^2-30^2) = 2100 \text{ ft-lb}$$

Consider the function $f(x) = \dfrac{2}{\sqrt{x}}$ on the interval $[1, 4]$. SCORE: ___ / 10 POINTS

[a] Find the value of c guaranteed by the Integral Mean Value Theorem. That is, find the appropriate value of c such that $f(c) = f_{ave}$.

$$f(c) = \frac{1}{4-1}\int_1^4 \frac{2}{\sqrt{x}}\,dx \quad 1$$

1

$$\frac{2}{\sqrt{c}} = \frac{1}{3}\left(4\sqrt{x}\right)\Big|_1^4$$

1

$$\frac{2}{\sqrt{c}} = \frac{4}{3}\left(\sqrt{4} - \sqrt{1}\right)$$

$$1 \quad \frac{2}{\sqrt{c}} = \frac{4}{3} \quad 1$$

$$6 = 4\sqrt{c}$$

1

$$\sqrt{c} = \frac{3}{2} \qquad c = \frac{9}{4}$$

[b] Write an integral for the length of the curve over the given interval.

$$\int_1^4 \sqrt{1 + \left(-x^{-\frac{3}{2}}\right)^2}\,dx = \int_1^4 \sqrt{1 + x^{-3}}\,dx$$

OR

OR $= \displaystyle\int_1^4 \sqrt{\frac{x^3+1}{x^3}}\,dx$

3 POINTS FOR ANY OF THESE

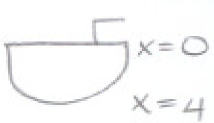
[c] Use your calculator's fnInt feature to find the length of the curve over the given interval to 2 decimal places.

3.21

1

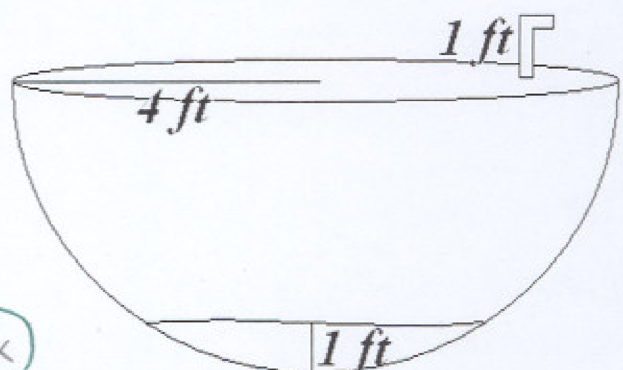
A hemispherical tank is filled 1 foot deep with water. Write, **BUT DO NOT EVALUATE**, an integral for the work required to pump the water out of the spout. Use 62.5 pounds per cubic foot for the density of water. SCORE: ___ / 8 POINTS

HINT: The total work is slightly less than 3125 **ft-lb.**



$$\int_3^4 62.5\,\pi\,(16-x^2)(x+1)\,dx$$

2 ½ 1 2 2 ½



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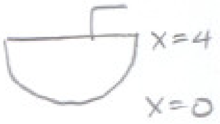
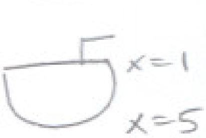


$$\int_0^1 62.5\,\pi\,(8x-x^2)(5-x)\,dx$$

2 ½ 1 2 2 ½

OR



2 ½ 1 2 2 ½

$$\int_4^5 62.5\,\pi\,(16-(x-1)^2)\,x\,dx$$