What month is your birthday ? ___ ___
What are the first 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___
What are the last 2 digits of your social security number ? ___ ___
[IF YOU DO NOT HAVE A SOCIAL SECURITY NUMBER,
USE YOUR STUDENT ID NUMBER]

SCORE: ___ / 30 POINTS

$TOTAL = 26\frac{1}{2}$

NO CALCULATORS ALLOWED
YOU MUST SHOW PROPER CALCULUS LEVEL WORK TO EARN FULL CREDIT

Find the length of the curve $y = \frac{1}{3}x(\sqrt{x}-3)$, $1 \le x \le 9$. **SCORE: ___ $4\frac{1}{2}$ POINTS**

$$y = \frac{1}{3}x^{\frac{3}{2}} - x$$

$$\frac{dy}{dx} = \frac{1}{2}x^{\frac{1}{2}} - 1 \quad \boxed{1\frac{1}{2}}$$

$$\int_1^9 \sqrt{1 + \left(\frac{1}{2}x^{\frac{1}{2}} - 1\right)^2}\, dx \quad \boxed{1\frac{1}{2}}$$

$$= \int_1^9 \sqrt{\frac{1}{4}x - x^{\frac{1}{2}} + 2}\, dx \quad \boxed{1\frac{1}{2}}$$

✳ COULD NOT BE COMPLETED
DUE TO TYPO IN QUESTION

Find the center of mass of the region between $y = x^3$ and $y = -x$ on the interval $[0, 2]$. **SCORE: ___ / 7 POINTS**

$$\int_0^2 (x^3 - -x)\, dx = \left(\frac{1}{4}x^4 + \frac{1}{2}x^2\right)\Big|_0^2 = 6 \quad \boxed{1} \quad \boxed{1}$$

$$\int_0^2 x(x^3 - -x)\, dx = \left(\frac{1}{5}x^5 + \frac{1}{3}x^3\right)\Big|_0^2 = \frac{32}{5} + \frac{8}{3} = \frac{136}{15} \quad \boxed{1} \quad \boxed{1}$$

$$\frac{1}{2}\int_0^2 \left((x^3)^2 - (-x)^2\right)\, dx = \frac{1}{2}\left(\frac{1}{7}x^7 - \frac{1}{3}x^3\right)\Big|_0^2 = \frac{1}{2}\left(\frac{128}{7} - \frac{8}{3}\right) = \frac{164}{21} \quad \boxed{1} \quad \boxed{1}$$

$$\text{CENTER OF MASS} = \left(\frac{\frac{136}{15}}{6}, \frac{\frac{164}{21}}{6}\right) = \left(\frac{136}{90}, \frac{164}{126}\right)$$

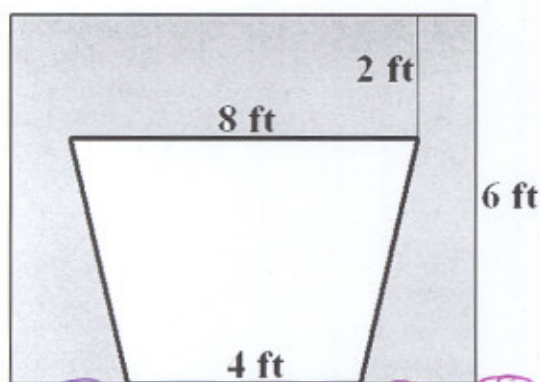$$= \left(\frac{68}{45}, \frac{82}{63}\right) \quad \boxed{\frac{1}{2}} \quad \boxed{\frac{1}{2}}$$

Find the hydrostatic force on the trapezoidal plate submerged in water. SCORE: ___ / 7 POINTS

NOTE: The bottom of the plate is 6 feet under the surface of the water. **You may use** ρ **as the density of water in your final answer.**

2 ft

8 ft

6 ft

4 ft

$x=0$ ①
$x=4$

$$\int_0^4 \rho\,(x+2)(8-x)\,dx$$

(1½) (1½)

$$= \rho\left(16x + 3x^2 - \tfrac{1}{3}x^3\right)\Big|_0^4$$

①

$$= \tfrac{272}{3}\rho$$

① ① ①

OR

$x=4$ ①
$x=0$

$$\int_0^4 \rho\,(6-x)(x+4)\,dx$$

(1½) (1½)

$$= \rho\left(24x + x^2 - \tfrac{1}{3}x^3\right)\Big|_0^4$$

①

$$= \tfrac{272}{3}\rho$$

① ①

OR

$x=2$ ①
$x=6$

$$\int_2^6 \rho\,(x)(10-x)\,dx$$

(1½) (1½)

$$= \rho\left(5x^2 - \tfrac{1}{3}x^3\right)\Big|_2^6$$

$$= \tfrac{272}{3}\rho$$

① ① ①

OR

$x=8$ ①
$x=4$

$$\int_4^8 \rho\,(10-x)(x)\,dx$$

(1½) (1½)

$$= \rho\left(5x^2 - \tfrac{1}{3}x^3\right)\Big|_4^8$$

$$= \tfrac{272}{3}\rho$$

① ① ①

Find the surface area if the curve $x = \sqrt[3]{y}$, $1 \le y \le 8$ is revolved around the x–axis. SCORE: ___ / 8 POINTS

$$y = x^3, \quad 1 \le x \le 2$$

① (½) ① ①

$$\int_1^2 2\pi x^3 \sqrt{1 + (3x^2)^2}\,dx$$

$$= \int_1^2 2\pi x^3 \sqrt{1 + 9x^4}\,dx$$

$u = 1 + 9x^4$
$\begin{cases} x=2 \longrightarrow u = 145 \\ x=1 \longrightarrow u = 10 \end{cases}$

$\dfrac{du}{dx} = 36x^3$

$\dfrac{1}{18}du = 2x^3\,dx$

(½)

$$= \int_{10}^{145} \tfrac{1}{18}\pi u^{\frac{1}{2}}\,du$$

②

$$= \tfrac{1}{27}\pi u^{\frac{3}{2}}\Big|_{10}^{145}$$

①

$$= \tfrac{\pi}{27}\left(145^{\frac{3}{2}} - 10^{\frac{3}{2}}\right)$$

①