Math 1B (9:30am – 10:20am)
Quiz 6 Version C
Fri May 21, 2010

SCORE: ___ / 30 POINTS

What month is your birthday ? ___ ___
What are the first 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___
What are the last 2 digits of your social security number ? ___ ___
[IF YOU DO NOT HAVE A SOCIAL SECURITY NUMBER, USE YOUR STUDENT ID NUMBER]

NO CALCULATORS ALLOWED

YOU MUST SHOW PROPER CALCULUS LEVEL WORK TO EARN FULL CREDIT

Find the center of mass of the region between $y = x^2$ and $y = -x$ on the interval $[1, 2]$. SCORE: ___ / 7 POINTS

$$\int_1^2 (x^2 - -x)\,dx = \left(\tfrac{1}{3}x^3 + \tfrac{1}{2}x^2\right)\Big|_1^2 = \tfrac{1}{3}(7) + \tfrac{1}{2}(3) = \tfrac{23}{6}$$ (1) (1)

$$\int_1^2 x(x^2 - -x)\,dx = \left(\tfrac{1}{4}x^4 + \tfrac{1}{3}x^3\right)\Big|_1^2 = \tfrac{1}{4}(15) + \tfrac{1}{3}(7) = \tfrac{73}{12}$$ (1) (1)

$$\tfrac{1}{2}\int_1^2 ((x^2)^2 - (-x)^2)\,dx = \tfrac{1}{2}\left(\tfrac{1}{5}x^5 - \tfrac{1}{3}x^3\right)\Big|_1^2 = \tfrac{1}{2}\left(\tfrac{1}{5}(31) - \tfrac{1}{3}(7)\right) = \tfrac{29}{15}$$ (1) (1)

$$\text{CENTER OF MASS} = \left(\frac{\frac{73}{12}}{\frac{23}{6}}, \frac{\frac{29}{15}}{\frac{23}{6}}\right) = \left(\tfrac{73}{46}, \tfrac{58}{115}\right)$$ ($\frac{1}{2}$) ($\frac{1}{2}$)

Find the surface area if the curve $y = \sqrt{x-1}$, $2 \le x \le 5$ is revolved around the x–axis. SCORE: ___ / 8 POINTS

(1) $\int_2^5$ ($\frac{1}{2}$) 2π (1) $\sqrt{x-1}$ (1) $\sqrt{1 + \left(\frac{1}{2\sqrt{x-1}}\right)^2}\,dx$

$$= \int_2^5 2\pi\sqrt{x-1}\sqrt{1 + \frac{1}{4(x-1)}}\,dx$$

$$= \int_2^5 2\pi\sqrt{x-1}\sqrt{\frac{4x-3}{4(x-1)}}\,dx$$

$$= \int_2^5 \underbrace{\pi\sqrt{4x-3}}_{(2)}\,dx$$

$u = 4x - 3$ ($x = 5 \to u = 17$, $x = 2 \to u = 5$)

$\frac{du}{dx} = 4$

$\frac{1}{4}du = dx$

$$= \int_5^{17} \tfrac{1}{4}\pi\sqrt{u}\,du$$

$$= \underbrace{\tfrac{1}{6}\pi u^{\frac{3}{2}}\Big|_5^{17}}_{(1)\ (\frac{1}{2})} = \underbrace{\tfrac{\pi}{6}\left(17^{\frac{3}{2}} - 5^{\frac{3}{2}}\right)}_{(1)}$$

☆ SEE OTHER SOLUTION IN 9:30 OTHER VERSION C QUESTION 4

Find the length of the curve $y = \dfrac{5x^8 + 3}{30x^3}$, $1 \le x \le 2$.

SCORE: ___ / 8 POINTS

$$y = \frac{1}{6}x^5 + \frac{1}{10}x^{-3}$$

$$\frac{dy}{dx} = \frac{5}{6}x^4 - \frac{3}{10}x^{-4} \quad (1\tfrac{1}{2})$$

$$\int_1^2 \sqrt{1 + \left(\frac{5}{6}x^4 - \frac{3}{10}x^{-4}\right)^2}\, dx \quad (1\tfrac{1}{2})$$

$$= \int_1^2 \sqrt{1 + \left(\frac{25}{36}x^8 - \frac{1}{2} + \frac{9}{100}x^{-8}\right)}\, dx$$

$$= \int_1^2 \sqrt{\frac{25}{36}x^8 + \frac{1}{2} + \frac{9}{100}x^{-8}}\, dx \quad (1\tfrac{1}{2})$$

$$= \int_1^2 \left(\frac{5}{6}x^4 + \frac{3}{10}x^{-4}\right) dx \quad (1\tfrac{1}{2})$$

$$= \left(\frac{1}{6}x^5 - \frac{1}{10}x^{-3}\right)\Big|_1^2 \quad (1)$$

$$= \frac{1}{6}(31) - \frac{1}{10}\left(\frac{1}{8} - 1\right)$$

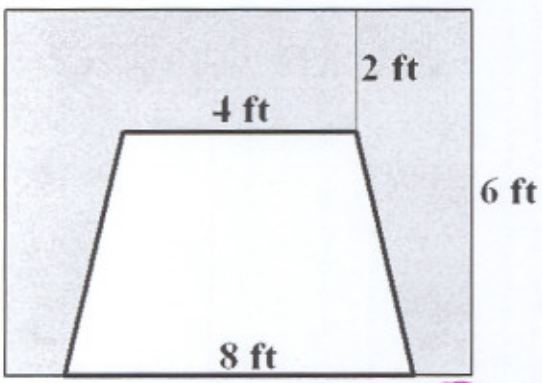
$$= \frac{31}{6} + \frac{7}{80}$$

$$= \frac{1240 + 21}{240} = \frac{1261}{240} \quad (1)$$

Find the hydrostatic force on the trapezoidal plate submerged in water.

SCORE: ___ / 7 POINTS

NOTE: The bottom of the plate is 6 feet under the surface of the water. **You may use ρ as the density of water in your final answer.**



$x=0$ to $x=4$: (1) $\int_0^4 \rho(x+2)(x+4)\, dx$ ($1\tfrac{1}{2}$) ($1\tfrac{1}{2}$)

$$= \rho\left(\frac{1}{3}x^3 + 3x^2 + 8x\right)\Big|_0^4 \quad (1)$$

$$= \frac{304}{3}\rho \quad (1)\ (1)$$

OR

$x=4$ to $x=0$: (1) $\int_0^4 \rho(6-x)(8-x)\, dx$ ($1\tfrac{1}{2}$) ($1\tfrac{1}{2}$)

$$= \rho\left(\frac{1}{3}x^3 - 7x^2 + 48x\right)\Big|_0^4 \quad (1)$$

$$= \frac{304}{3}\rho \quad (1)\ (1)$$

OR

$x=2$ to $x=6$: (1) $\int_2^6 \rho x(x+2)\, dx$ ($1\tfrac{1}{2}$) ($1\tfrac{1}{2}$)

$$= \rho\left(\frac{1}{3}x^3 + x^2\right)\Big|_2^6 \quad (1)$$

$$= \frac{304}{3}\rho \quad (1)\ (1)$$

OR

$x=4$ to $x=8$: (1) $\int_4^8 \rho(x-2)x\, dx$ ($1\tfrac{1}{2}$) ($1\tfrac{1}{2}$)

$$= \rho\left(\frac{1}{3}x^3 - x^2\right)\Big|_4^8 \quad (1)$$

$$= \frac{304}{3}\rho \quad (1)$$