

NO CALCULATORS ALLOWEDYOU MUST SHOW PROPER CALCULUS LEVEL WORK
(INCLUDING SANITY CHECKS WHERE RELEVANT) TO EARN FULL CREDIT

Find $\int \frac{1}{x^2 \sqrt{x^2 - 9}} dx$.

SCORE: ___ / 7 POINTS

$$\textcircled{1} \quad x = 3 \sec \theta \quad \longrightarrow \quad \sec \theta = \frac{x}{3}$$

$$dx = 3 \sec \theta \tan \theta d\theta$$



$$\int \frac{1}{9 \sec^2 \theta \sqrt{9 \sec^2 \theta - 9}} 3 \sec \theta \tan \theta d\theta \quad \textcircled{1}$$

$$= \int \frac{1}{9 \sec^2 \theta (3 \tan \theta)} 3 \sec \theta \tan \theta d\theta \quad \textcircled{1}$$

$$= \frac{1}{9} \int \cos \theta d\theta \quad \textcircled{1}$$

$$= \frac{1}{9} \sin \theta + C$$

$$= \frac{1}{9} \frac{\sqrt{x^2 - 9}}{x} + C$$

FOR EACH QUESTION:
SUBTRACT $\frac{1}{2}$ IF MISSING
" + C "SUBTRACT $\frac{1}{2}$ FOR EACH
ABSOLUTE VALUE
MISSING

Find $\int \frac{x+5}{(x+1)(x-1)^2} dx$.

SCORE: ___ / 7 POINTS

$$= \int \left(\frac{A}{x+1} + \frac{B}{x-1} + \frac{C}{(x-1)^2} \right) dx$$

$$= \int \left(\frac{1}{x+1} - \frac{1}{x-1} + \frac{3}{(x-1)^2} \right) dx$$

$$= \underbrace{\ln|x+1|}_{\textcircled{\frac{1}{2}}} - \underbrace{\ln|x-1|}_{\textcircled{\frac{1}{2}}} - \underbrace{\frac{3}{x-1}}_{\textcircled{1}} + C$$

$$x+5 = A(x-1)^2 + B(x+1)(x-1) + C(x+1)$$

$$x=1: 6 = 2C \rightarrow C=3$$

$$x=-1: 4 = 4A \rightarrow A=1$$

$$x=0: 5 = A - B + C$$

$$5 = 1 - B + 3$$

$$B = -1$$

$$\textcircled{1} \quad \text{SANITY CHECK:}$$

$$x=2: \text{ORIGINAL} = \frac{7}{3}$$

$$\text{DECOMPOSITION} = \frac{1}{3} + \frac{-1}{1} + \frac{3}{1} = \frac{7}{3}$$

Find $\int x(\ln x)^2 dx$.

SCORE: 6 / 6 POINTS

$$\begin{array}{r} \frac{u}{(ln x)^2} \quad \frac{dv}{+x} \\ 2ln x \cdot \frac{1}{x} \quad - \frac{1}{2} x^2 \\ \hline 2ln x \quad - \frac{1}{2} x \\ \frac{2}{x} \quad - \frac{1}{4} x^2 \\ \hline 2 \quad + \frac{1}{4} x \\ 0 \quad - \frac{1}{8} x^2 \end{array}$$

$$\frac{1}{2} x^2 (\ln x)^2 - \frac{1}{2} x^2 \ln x + \frac{1}{4} x^2 + C$$

(3) (2) (1)

Find $\int e^{3x} \cos 4x dx$.

= $\frac{1}{4} e^{3x} \sin 4x$ (1/2) + $\frac{3}{16} e^{3x} \cos 4x$ (1/2) - $\frac{9}{16} \int e^{3x} \cos 4x dx$ (1) SCORE: 6 / 6 POINTS

$$\begin{array}{r} \frac{u}{e^{3x}} \quad \frac{dv}{+ \cos 4x} \\ 3e^{3x} \quad - \frac{1}{4} \sin 4x \\ 9e^{3x} \quad - \frac{1}{16} \cos 4x \end{array}$$

so,

$$\frac{25}{16} \int e^{3x} \cos 4x dx = \frac{1}{4} e^{3x} \sin 4x + \frac{3}{16} e^{3x} \cos 4x$$

(1)

SEE ALSO
7:30 VERSION B

$$\int e^{3x} \cos 4x dx = \frac{4}{25} e^{3x} \sin 4x + \frac{3}{25} e^{3x} \cos 4x + C$$

(1/2) (1/2)

Find $\int \frac{10-x^2}{x^2+2x-8} dx$.

SCORE: 4 / 4 POINTS

$$\begin{aligned} &= \int \left(-1 + \frac{2x+2}{x^2+2x-8} \right) dx \\ &= -x + \ln |x^2+2x-8| + C \end{aligned}$$

(1/2) (1/2)

$$\begin{array}{r} -1 \\ x^2+2x-8 \overline{) -x^2 \quad +10} \\ \hline -x^2-2x+8 \\ \hline 2x+2 \end{array}$$