

NO CALCULATORS ALLOWED**YOU MUST SHOW PROPER CALCULUS LEVEL WORK
(INCLUDING SANITY CHECKS WHERE RELEVANT) TO EARN FULL CREDIT**Find $\int \frac{e^{2x}}{1+e^x} dx$.

SCORE: ___ / ___ POINTS

$$\begin{aligned}
 u &= 1+e^x \\
 du &= e^x dx \\
 \frac{1}{e^x} du &= dx \\
 \frac{e^{2x}}{1+e^x} \cdot \frac{1}{e^x} du &= \frac{e^x}{1+e^x} du = \frac{u-1}{u} du \\
 \int \frac{u-1}{u} du &= \int (1 - \frac{1}{u}) du \\
 &= u - \ln|u| + C \\
 &= 1+e^x - \ln|1+e^x| + C \\
 &= 1+e^x - \ln(1+e^x) + C
 \end{aligned}$$



SEE 7:30
VERSION B
QUESTION 4
FOR
ALTERNATE
SOLUTION

Evaluate $\int_0^3 (x-1)^{-3} dx$.

SCORE: ___ / ___ POINTS

$$= \int_0^1 (x-1)^{-3} dx + \int_1^3 (x-1)^{-3} dx$$

$$\hookrightarrow = \lim_{N \rightarrow 1^-} \int_0^N (x-1)^{-3} dx \quad \begin{array}{l} u = x-1 \\ du = dx \end{array} \quad \begin{array}{l} x = N \rightarrow u = N-1 \\ x = 0 \rightarrow u = -1 \end{array}$$

$$= \lim_{N \rightarrow 1^-} \int_{-1}^{N-1} u^{-3} du$$

$$= \lim_{N \rightarrow 1^-} \left. -\frac{1}{2} u^{-2} \right|_{-1}^{N-1}$$

$$= \lim_{N \rightarrow 1^-} -\frac{1}{2} ((N-1)^{-2} - 1) \rightarrow -\infty$$

DIVERGES

Find $\int \tan^{-1} \sqrt{x} \, dx$.

SCORE: ___ / ___ POINTS

$$u = \sqrt{x}$$

$$du = \frac{1}{2\sqrt{x}} dx$$

$$2\sqrt{x} du = dx$$

$$\tan^{-1} \sqrt{x} \cdot 2\sqrt{x} du = 2u \tan^{-1} u du \quad \text{BY PARTS}$$

$$\int 2u \tan^{-1} u du = u^2 \tan^{-1} u - \int \frac{u^2}{1+u^2} du \quad \begin{array}{l} \frac{u}{\tan^{-1} u} \frac{du}{+2u} \\ \frac{1}{1+u^2} = \frac{1}{u^2} \end{array}$$

$$= u^2 \tan^{-1} u - \int \left(1 - \frac{1}{1+u^2}\right) du$$

$$= u^2 \tan^{-1} u - u + \tan^{-1} u + C$$

$$= x \tan^{-1} \sqrt{x} - \sqrt{x} + \tan^{-1} \sqrt{x} + C$$

Evaluate $\int_2^{\infty} x e^{-\frac{x}{2}} dx$.

SCORE: ___ / ___ POINTS

$$= \lim_{N \rightarrow \infty} \int_2^N x e^{-\frac{x}{2}} dx$$

$$= \lim_{N \rightarrow \infty} \left(-2x e^{-\frac{x}{2}} - 4e^{-\frac{x}{2}} \right) \Big|_2^N$$

$$= \lim_{N \rightarrow \infty} (-2N e^{-\frac{N}{2}} - 4e^{-\frac{N}{2}} - (-4e^{-1} - 4e^{-1}))$$

$$= 0 - 0 + 4e^{-1} + 4e^{-1}$$

$$= 8e^{-1}$$

$$\lim_{N \rightarrow \infty} -2N e^{-\frac{N}{2}} \quad -\infty \cdot 0$$

$$= \lim_{N \rightarrow \infty} \frac{-2N}{e^{\frac{N}{2}}} \quad \frac{-\infty}{\infty}$$

$$= \lim_{N \rightarrow \infty} \frac{-2}{\frac{1}{2} e^{\frac{N}{2}}} \quad \frac{-2}{\infty}$$

$$= 0$$

HAND IN THIS SECTION TO RECEIVE THE TABLE OF INTEGRALS

What month is your birthday? _____

What are the first 2 digits of your address? _____

What are the last 2 digits of your zip code? _____

What are the last 2 digits of your social security number? _____

[IF YOU DO NOT HAVE A SOCIAL SECURITY NUMBER,
USE YOUR STUDENT ID NUMBER]

SCORE: ____ / 30 POINTS

NO CALCULATORS ALLOWED**YOU MUST SHOW PROPER CALCULUS LEVEL WORK
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SCORE: ____ / ____ POINTS

STATE THE NUMBER OF THE RULE YOU ARE USING.

$$u = x^3$$

$$du = 3x^2 dx$$

$$\frac{1}{3x^2} du = dx$$

$$\frac{1}{x^4 \sqrt{x^6+4}} \cdot \frac{1}{3x^2} du = \frac{1}{3x^6 \sqrt{x^6+4}} du = \frac{1}{3} \frac{1}{u^2 \sqrt{u^2+4}} du$$

$$\frac{1}{3} \int \frac{1}{u^2 \sqrt{u^2+4}} du = -\frac{1}{3} \frac{\sqrt{4+u^2}}{4u} + C = -\frac{\sqrt{x^6+4}}{12x^3} + C$$

RULE (28) $a=2$ **USE THE SPACE BELOW AND ON THE OTHER SIDE FOR SCRATCH WORK**