

What month is your birthday?

What are the first 2 digits of your address?

What are the last 2 digits of your zip code?

What are the last 2 digits of your social security number?

[IF YOU DO NOT HAVE A SOCIAL SECURITY NUMBER,
USE YOUR STUDENT ID NUMBER]

SCORE: ___ / 30 POINTS

NO CALCULATORS ALLOWED**YOU MUST SHOW PROPER CALCULUS LEVEL WORK
(INCLUDING SANITY CHECKS WHERE RELEVANT) TO EARN FULL CREDIT**Find $\int x^2(x-1)^{-4} dx$.

SCORE: ___ / ___ POINTS

$$u = x - 1$$

$$du = dx$$

$$\begin{aligned} \int x^2(x-1)^{-4} dx &= \int (u+1)^2 u^{-4} du \\ &= \int (u^2 + 2u + 1) u^{-4} du \\ &= \int (u^{-2} + 2u^{-3} + u^{-4}) du \\ &= -u^{-1} - u^{-2} - \frac{1}{3}u^{-3} + C \\ &= -\frac{1}{x-1} - \frac{1}{(x-1)^2} - \frac{1}{3(x-1)^3} + C \end{aligned}$$

★
SEE 9:30 VERSION D
QUESTION 4 FOR
ALTERNATE
SOLUTION

Evaluate $\int_0^5 (x-2)^{-4} dx$.

SCORE: ___ / ___ POINTS

$$= \int_0^2 (x-2)^{-4} dx + \int_2^5 (x-2)^{-4} dx$$

$$\hookrightarrow = \lim_{N \rightarrow 2^-} \int_0^N (x-2)^{-4} dx$$

$$u = x - 2 \begin{cases} x = N \rightarrow u = N - 2 \\ x = 0 \rightarrow u = -2 \end{cases}$$

$$du = dx$$

$$= \lim_{N \rightarrow 2^-} \int_{-2}^{N-2} u^{-4} du$$

$$= \lim_{N \rightarrow 2^-} \left. -\frac{1}{3} u^{-3} \right|_{-2}^{N-2}$$

$$= \lim_{N \rightarrow 2^-} -\frac{1}{3} \left((N-2)^{-3} + \frac{1}{8} \right) \rightarrow \infty$$

DIVERGES

Find $\int \frac{1}{1+e^x} dx$.

SCORE: ___ / ___ POINTS

★ SEE 9:30 VERSION D QUESTION 2
FOR ALTERNATE SOLUTIONS

$$u = 1 + e^x$$

$$du = e^x dx$$

$$\frac{1}{e^x} du = dx$$

$$\frac{1}{1+e^x} \frac{1}{e^x} du = \frac{1}{u(u-1)} du$$

PARTIAL
FRACTIONS

$$\frac{1}{u(u-1)} = \frac{A}{u} + \frac{B}{u-1}$$

$$1 = A(u-1) + Bu$$

$$u=1: 1 = B$$

$$u=0: 1 = -A \Rightarrow A = -1$$

CHECK

$$u=-1: \frac{1}{(-1)(-2)} = \frac{1}{2}$$

$$\frac{-1}{-1} + \frac{1}{-2} = 1 - \frac{1}{2} = \frac{1}{2} \checkmark$$

$$\int \frac{1}{u(u-1)} du = \int \left(\frac{-1}{u} + \frac{1}{u-1} \right) du$$

$$= -\ln|u| + \ln|u-1| + C$$

$$= -\ln|1+e^x| + \ln|e^x| + C$$

$$= x - \ln(1+e^x) + C$$

Evaluate $\int_3^{\infty} x e^{-\frac{x}{3}} dx$.

SCORE: ___ / ___ POINTS

$$= \lim_{N \rightarrow \infty} \int_3^N x e^{-\frac{x}{3}} dx$$

$$\begin{array}{l} u \\ x \end{array} \quad \begin{array}{l} dv \\ e^{-\frac{x}{3}} \end{array}$$

$$\begin{array}{l} 1 \\ 0 \end{array} \quad \begin{array}{l} -3e^{-\frac{x}{3}} \\ 9e^{-\frac{x}{3}} \end{array}$$

$$= \lim_{N \rightarrow \infty} \left(-3x e^{-\frac{x}{3}} - 9e^{-\frac{x}{3}} \right) \Big|_3^N$$

$$= \lim_{N \rightarrow \infty} \left(-3N e^{-\frac{N}{3}} - 9e^{-\frac{N}{3}} - (-9e^{-1} - 9e^{-1}) \right)$$

$$= 0 - 0 + 9e^{-1} + 9e^{-1}$$

$$= 18e^{-1}$$

$$\lim_{N \rightarrow \infty} -3N e^{-\frac{N}{3}} \quad -\infty \cdot 0$$

$$= \lim_{N \rightarrow \infty} \frac{-3N}{e^{\frac{N}{3}}} \quad \frac{-\infty}{\infty}$$

$$= \lim_{N \rightarrow \infty} \frac{-3}{\frac{1}{3} e^{\frac{N}{3}}} \quad \frac{-3}{\infty}$$

$$= 0$$

HAND IN THIS SECTION TO RECEIVE THE TABLE OF INTEGRALS

What month is your birthday ?

What are the first 2 digits of your address ?

What are the last 2 digits of your zip code ?

What are the last 2 digits of your social security number ?

[IF YOU DO NOT HAVE A SOCIAL SECURITY NUMBER,
USE YOUR STUDENT ID NUMBER]

SCORE: ___ / 30 POINTS

NO CALCULATORS ALLOWED**YOU MUST SHOW PROPER CALCULUS LEVEL WORK
(INCLUDING SANITY CHECKS WHERE RELEVANT) TO EARN FULL CREDIT**Evaluate $\int \frac{1}{\sqrt{e^{4x}+9}} dx$ using the table of integrals.

SCORE: ___ / ___ POINTS

STATE THE NUMBER OF THE RULE YOU ARE USING.

$$u = e^{2x}$$

$$du = 2e^{2x} dx$$

$$\frac{1}{2e^{2x}} du = dx$$

$$\frac{1}{\sqrt{e^{4x}+9}} \frac{1}{2e^{2x}} du = \frac{1}{2} \frac{1}{e^{2x} \sqrt{e^{4x}+9}} du = \frac{1}{2} \frac{1}{u \sqrt{u^2+9}} du$$

$$\frac{1}{2} \int \frac{1}{u \sqrt{u^2+9}} du = -\frac{1}{2} \frac{1}{3} \ln \left| \frac{\sqrt{u^2+9}+3}{u} \right| + C$$

RULE (27) $a=3$

$$= -\frac{1}{6} \ln \left| \frac{\sqrt{e^{4x}+9}+3}{e^{2x}} \right| + C$$

$$= -\frac{1}{6} \ln (\sqrt{e^{4x}+9}+3) + \frac{1}{3}x + C$$

USE THE SPACE BELOW AND ON THE OTHER SIDE FOR SCRATCH WORK