Math 114
Midterm 1
Thu Jan 28, 2010

Name: _____

SCORE: _____ / 140 POINTS

- ALL PROBLEMS MUST BE SOLVED ALGEBRAICALLY TO EARN CREDIT
- PUT A BOX AROUND EACH FINAL ANSWER
- SHOW COMPLETE AND PROPER WORK TO EARN FULL CREDIT

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Multiply and simplify: $\dfrac{x^2-16}{x^2+5x-6}\cdot\dfrac{3x^2+12x-36}{x^2+2x-8}$

SCORE: ___ / 10 POINTS

$$= \frac{(x+4)(x-4)}{(x+6)(x-1)}\cdot\frac{3(x^2+4x-12)}{(x+4)(x-2)} = \frac{(x+4)(x-4)}{(x+6)(x-1)}\cdot\frac{3(x+6)(x-2)}{(x+4)(x-2)} \star$$

$$= \boxed{\frac{3(x-4)}{x-1}} \star$$

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Subtract and simplify: $\dfrac{x+3}{x^2-2x-3}-\dfrac{x+1}{x^2-6x+9}$

SCORE: ___ / 12 POINTS

$$= \frac{x+3}{(x-3)(x+1)} - \frac{x+1}{(x-3)^2} = \frac{x+3}{(x-3)(x+1)}\cdot\frac{x-3}{x-3} - \frac{x+1}{(x-3)^2}\cdot\frac{x+1}{x+1} \star$$

$$= \frac{x^2-9-(x^2+2x+1)}{(x-3)^2(x+1)} \star = \frac{-2x-10}{(x-3)^2(x+1)} = \boxed{\frac{-2(x+5)}{(x-3)^2(x+1)}} \star$$

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Write a proportion for, but **DO NOT SOLVE**:

SCORE: ___ / 4 POINTS

If 225 US dollars is equivalent to 155 Euros, how many Euros is 140 US dollars equivalent to ?

$$\boxed{\frac{225}{155} = \frac{140}{x}} \star$$

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Add and simplify: $\dfrac{4}{x+2}+\dfrac{x-3}{x-1}$

SCORE: ___ / 10 POINTS

$$= \frac{4}{x+2}\cdot\frac{x-1}{x-1} + \frac{x-3}{x-1}\cdot\frac{x+2}{x+2}$$

$$= \frac{4x-4+x^2-x-6}{(x+2)(x-1)} \star = \frac{x^2+3x-10}{(x+2)(x-1)} \star = \boxed{\frac{(x+5)(x-2)}{(x+2)(x-1)}} \star$$

Fill in the blanks: The equation of the horizontal asymptote for $y = \dfrac{4-7x}{13x+8}$ is $y = -\dfrac{7}{13}$ ★. $y \approx \dfrac{-7x}{13x} = -\dfrac{7}{13}$ **SCORE: ___ / 10 POINTS**

The equation of the vertical asymptote for $y = \dfrac{4-7x}{13x+8}$ is $x = -\dfrac{8}{13}$ ★. $13x+8=0$, $x=\dfrac{-8}{13}$

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Simplify: $\dfrac{\frac{3}{x}+\frac{2}{x-3}}{\frac{4}{x-3}-\frac{5}{x}}$ **SCORE: ___ / 10 POINTS**

$$= \frac{\frac{3}{x}+\frac{2}{x-3}}{\frac{4}{x-3}-\frac{5}{x}} \cdot \frac{x(x-3)}{x(x-3)} = \frac{3(x-3)+2x}{4x-5(x-3)} \text{ ★} = \frac{3x-9+2x}{4x-5x+15}$$

$$= \boxed{\frac{5x-9}{-x+15}} \text{ ★}$$

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Subtract and simplify: $\dfrac{2x^2-2x+7}{x^2-x-12} - \dfrac{x^2+5x-5}{x^2-x-12}$ **SCORE: ___ / 10 POINTS**

$$= \frac{x^2-7x+12}{x^2-x-12} \text{ ★}$$

$$= \frac{(x-3)(x-4)}{(x-4)(x+3)} \text{ ★} = \boxed{\frac{x-3}{x+3}} \text{ ★}$$

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Simplify: $\dfrac{\frac{3}{x+2}-4}{6+\frac{5}{x+2}}$ **SCORE: ___ / 10 POINTS**

$$= \frac{\frac{3}{x+2}-4}{6+\frac{5}{x+2}} \cdot \frac{x+2}{x+2} = \frac{3-4(x+2)}{6(x+2)+5} = \frac{3-4x-8}{6x+12+5} \text{ ★}$$

$$= \boxed{\frac{-4x-5}{6x+17}} \text{ ★}$$

Divide and simplify: $\dfrac{4x^2-12x}{4x^2-1} \div \dfrac{2x^2-18}{6x+3}$

**SCORE: ___ / 10 POINTS**

$= \dfrac{4x^2-12x}{4x^2-1} \cdot \dfrac{6x+3}{2x^2-18} = \dfrac{4x(x-3)}{(2x+1)(2x-1)} \cdot \dfrac{3(2x+1)}{2(x^2-9)}$

$= \dfrac{2\,4x(x-3)}{(2x+1)(2x-1)} \cdot \dfrac{3(2x+1)}{2(x+3)(x-3)}$ ★

$= \boxed{\dfrac{6x}{(2x-1)(x+3)}}$ ★

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Simplify: $\dfrac{4x^2+3x-10}{3x^2+10x+8}$

**SCORE: ___ / 10 POINTS**

$= \dfrac{(4x-5)(x+2)}{(3x+4)(x+2)}$ ★

$= \boxed{\dfrac{4x-5}{3x+4}}$ ★

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Solve: A number divided by nine is equal to two divided by three less than that number. Find the number. **CHECK YOUR ANSWER(S).**

**SCORE: ___ / 10 POINTS**

$\dfrac{x}{9} = \dfrac{2}{x-3}$ ★

$x^2-3x=18$

$x^2-3x-18=0$ ★

$(x-6)(x+3)=0$ ★

$x-6=0$ or $x+3=0$

$\boxed{x=6}$ or $\boxed{x=-3}$ ★

CHECK: $\dfrac{6}{9} \stackrel{?}{=} \dfrac{2}{6-3}$ ★

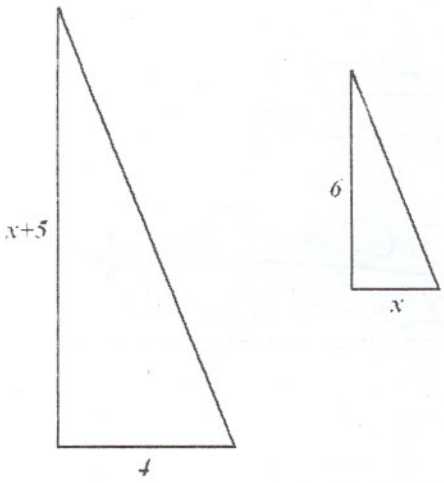
$\dfrac{2}{3} = \dfrac{2}{3}$ ✓ ★

$\dfrac{-3}{9} \stackrel{?}{=} \dfrac{2}{-3-3}$ ★

$-\dfrac{1}{3} \stackrel{?}{=} \dfrac{2}{-6}$

$-\dfrac{1}{3} = -\dfrac{1}{3}$ ✓ ★

Solve for $x$ in the following similar triangles: **SCORE: ___ / 10 POINTS**




$$\frac{x+5}{6} = \frac{4}{x}$$

$$x^2+5x = 24$$

$$x^2+5x-24=0$$

$$(x+8)(x-3)=0$$

$$x+8=0 \text{ OR } x-3=0$$

$$x=-8 \text{ OR } \boxed{x=3}$$

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Solve: The number of workers required to complete a job varies directly as the number of pieces that must be manufactured and inversely as the number of hours allocated for the job. If 12 workers are needed to manufacture 750 pieces in 6 hours, how many workers are needed to manufacture 1000 pieces in 16 hours? **SCORE: ___ / 12 POINTS**

**FOR FULL CREDIT, YOU MUST IDENTIFY WHAT ALL YOUR VARIABLES REPRESENT, FIND THE EQUATIONS CONNECTING THEM, AND SUMMARIZE YOUR FINAL ANSWER.**

LET $w$ = # WORKERS

$p$ = # PIECES

$h$ = # HOURS

$$w = \frac{kp}{h}$$

$$12 = \frac{k(750)}{6}$$

$$72 = 750k$$

$$k = 0.096$$

$$w = \frac{0.096p}{h}$$

$$w = \frac{0.096(1000)}{16}$$

$$= 6$$

6 WORKERS ARE NEEDED TO MANUFACTURE 1000 PIECES IN 16 HOURS

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Solve for $x$: $\frac{1}{2x}+\frac{3}{x^2}=\frac{1}{6}$ **CHECK YOUR ANSWER(S)** **SCORE: ___ / 12 POINTS**

$$6x^2\left(\frac{1}{2x}+\frac{3}{x^2}\right)=\left(\frac{1}{6}\right)6x^2$$

$$3x+18=x^2$$

$$0=x^2-3x-18$$

$$0=(x-6)(x+3)$$

$$x-6=0 \text{ OR } x+3=0$$

$$\boxed{x=6} \text{ OR } \boxed{x=-3}$$

CHECK: $\frac{1}{2(6)}+\frac{3}{6^2}$ $\quad$ $\frac{1}{2(-3)}+\frac{3}{(-3)^2}$

$= \frac{1}{12}+\frac{3}{36}$ $\quad$ $= -\frac{1}{6}+\frac{3}{9}$

$= \frac{1}{12}+\frac{1}{12}$ $\quad$ $= -\frac{1}{6}+\frac{1}{3}$

$= \frac{2}{12}$ $\quad$ $= -\frac{1}{6}+\frac{2}{6}$

$= \frac{1}{6}$ ✓ $\quad$ $= \frac{1}{6}$ ✓