

SCORE: ___ / 10 POINTS

Find all discontinuities for the following function. Determine whether each discontinuity is removable, jump or infinite. Show proper algebraic work and write using proper notation as shown in class.

SCORE: ___ / 4 POINTS

$$f(x) = \begin{cases} 1 - \cos x & \text{if } x < 0 \\ \tan x & \text{if } 0 < x < \pi \\ \sin x & \text{if } x \geq \pi \end{cases}$$

NOTE: $x = \pi$ IS NOT A DISCONTINUITY, SINCE $\lim_{x \rightarrow \pi} f(x) = f(\pi) = 0$

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} (1 - \cos x) = 0$$

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \tan x = 0$$

$$\lim_{x \rightarrow 0} f(x) = 0 \text{ BUT } f(0) \text{ DNE}$$

SO, $x = 0$ IS A REMOVABLE DISCONTINUITY

$$\lim_{x \rightarrow \frac{\pi}{2}^-} f(x) = \lim_{x \rightarrow \frac{\pi}{2}^-} \tan x = \lim_{x \rightarrow \frac{\pi}{2}^-} \frac{\sin x}{\cos x}$$

$$\text{AS } x \rightarrow \frac{\pi}{2}^-, \sin x \rightarrow 1, \cos x \rightarrow 0^+$$

$$\text{SO, } \tan x \rightarrow \infty$$

SO, $x = \frac{\pi}{2}$ IS AN INFINITE DISCONTINUITY

Find all asymptotes for the following function. For each vertical asymptote, determine both one-sided limits. Show proper algebraic work and write using proper notation as shown in class.

SCORE: ___ / 3 POINTS

$$f(x) = \frac{11 - 7(0.2^x)}{17(0.2^x) + 13}$$

$$\lim_{x \rightarrow \infty} \frac{11 - 7(0.2^x)}{17(0.2^x) + 13} = \frac{11 - 7(0)}{17(0) + 13} = \frac{11}{13}$$

$$\lim_{x \rightarrow -\infty} \frac{11 - 7(0.2^x)}{17(0.2^x) + 13} \cdot \frac{5^x}{5^x} = \lim_{x \rightarrow -\infty} \frac{11(5^x) - 7}{17 + 13(5^x)} = \frac{0 - 7}{17 + 0} = -\frac{7}{17}$$

HORIZONTAL ASYMPTOTES $y = \frac{11}{13}$ AND $y = -\frac{7}{17}$

NO VERTICAL ASYMPTOTES SINCE $0.2^x > 0$, SO $17(0.2^x) + 13 > 0$

Prove that the equation $\tan^{-1} x = x$ has a solution in the interval $[-1, 1]$. Show proper justification.

SCORE: ___ / 3 POINTS

$$\text{LET } f(x) = \tan^{-1} x - x$$

SINCE $\tan^{-1} x$ AND x ARE CONTINUOUS, SO IS $\tan^{-1} x - x$

$$f(-1) = \tan^{-1}(-1) - (-1) = -\frac{\pi}{4} + 1 > 0$$

$$f(1) = \tan^{-1} 1 - 1 = \frac{\pi}{4} - 1 < 0$$

$$f(1) < 0 < f(-1)$$

BY IVT, THERE IS A VALUE $c \in (-1, 1)$ SUCH THAT $\tan^{-1} c - c = 0$
IE. $\tan^{-1} c = c$