[1] Determine whether the Extreme Value Theorem, Rolle's Theorem and/or the Mean Value Theorem apply to the given function on the given interval. You must justify your answer – which hypotheses of each theorem are satisfied and why, which hypotheses are not satisfied and why not.
NOTE: None, one, two or all three theorems may apply in each case.

[a] $f(x) = x^{\frac{2}{3}}$ on $[-1, 1]$

[b] $f(x) = \tan x$ on $\left[-\dfrac{\pi}{4}, \dfrac{\pi}{4}\right]$

[c] $f(x) = \sec x$ on $[-\pi, \pi]$

[d] $f(x) = x^{\frac{4}{3}}$ on $[-1, 1]$

[2] Draw a function that satisfies the following conditions.

[a] f' is never positive or zero on $[0, 5]$,
but f is not decreasing on $[0, 5]$.

[b] $\lim\limits_{x \to 0} f(x) = f(0)$,

$\lim\limits_{h \to 0} \dfrac{f(h) - f(0)}{h} \neq 0$,

$x = 0$ is a critical number of f,

f does not have a local extrema nor a cusp at $x = 0$.

[3] Draw a continuous function that satisfies the following conditions.

[a] $f'(x) > 0$ and $f''(x) > 0$ on $[-4, -2]$,
$f'(x) < 0$ and $f''(x) < 0$ on $[-2, 0]$,
$f'(x) > 0$ and $f''(x) < 0$ on $[0, 2]$,
and $f'(x) < 0$ and $f''(x) > 0$ on $[2, 4]$.

[b] $f'(x) < 0$ and $f''(x) > 0$ on $[-4, -2]$,
$f'(x) > 0$ and $f''(x) < 0$ on $[-2, 0]$,
$f'(x) > 0$ and $f''(x) > 0$ on $[0, 2]$,
and $f'(x) < 0$ and $f''(x) < 0$ on $[2, 4]$.