Math 1A (9:30am – 10:20am)
Midterm 1
Thu Jan 28, 2009

$* = 100$ POINTS
$> 70\%$

SCORE: ___ / 140 POINTS

What month is your birthday ? __ __
What are the first 2 digits of your address ? __ __
What are the last 2 digits of your zip code ? __ __
What are the last 2 digits of your social security number ? __ __
[IF YOU DO NOT HAVE A SOCIAL SECURITY NUMBER, USE YOUR STUDENT ID NUMBER]

☆ NO CALCULATORS ALLOWED
☆ YOU MUST SHOW PROPER CALCULUS-LEVEL WORK AND LOGIC

State the definition of a vertical asymptote. **SCORE: * / 5 POINTS**

f HAS A VERTICAL ASYMPTOTE AT $x=a$ IF $\lim_{x\to a^+} f(x) = \infty$ OR $\lim_{x\to a^+} f(x) = -\infty$ OR $\lim_{x\to a^-} f(x) = \infty$ OR $\lim_{x\to a^-} f(x) = -\infty$

State the Intermediate Value Theorem. **SCORE: * / 5 POINTS**

IF f IS CONT ON $[a,b]$ AND $f(a) < d < f(b)$ OR $f(b) < d < f(a)$

THEN THERE IS A $c \in (a,b)$ SUCH THAT $f(c) = d$

State the definition of continuity at a point. **SCORE: * / 5 POINTS**

f IS CONT. @ $x=a$ IF $f(a)$ AND $\lim_{x\to a} f(x)$ BOTH EXIST AND ARE EQUAL

Find the equation of the tangent line to the curve of $f(x) = \dfrac{x}{1-2x}$ at $x = 1$. **SCORE: * / 14 POINTS**

$$f'(1) = \lim_{x\to 1} \frac{\frac{x}{1-2x} - (-1)}{x-1} = \lim_{x\to 1} \frac{x+1-2x}{(x-1)(1-2x)} = \lim_{x\to 1} \frac{\cancel{1-x}\ {-1}}{\cancel{(x-1)}(1-2x)} = 1$$

$$y - (-1) = 1(x-1)$$
$$y + 1 = x - 1$$
$$y = x - 2$$

Your score on a test depends on how much time you spent studying for it the 24 hours just before the test. **SCORE: ____ / 12 POINTS**
Let $P = f(s)$, where P is the number of points you score on a test, and s is the number of hours you studied the day before.

[a] Give the (practical) meaning, including units, for the algebraic statement $f'(4) = 7$.

IF YOU STUDY FOR 4 HOURS THE DAY BEFORE A TEST, YOUR SCORE WOULD GO UP 7 POINTS FOR EACH ADDITIONAL HOUR OF STUDYING.

[b] Is there a value s_0 for which you would expect $f'(s_0) < 0$? Why or why not ?

YES. IF YOU STUDY TOO MUCH AND DON'T GET ENOUGH SLEEP, YOUR SCORE WILL GO DOWN INSTEAD.

The position of an object (in feet) at time t minutes, is given by the function $f(t) = \sqrt{t^2 + 5}$. **SCORE: * / 14 POINTS**
Find the instantaneous velocity of the object at time $t = 2$. Specify the units.

$$f'(2) = \lim_{h \to 0} \frac{\sqrt{(2+h)^2+5} - 3}{h} = \lim_{h \to 0} \frac{\sqrt{h^2+4h+9} - 3}{h} \cdot \frac{\sqrt{h^2+4h+9} + 3}{\sqrt{h^2+4h+9} + 3}$$

$$= \lim_{h \to 0} \frac{h^2 + 4\cancel{h}}{\cancel{h}(\sqrt{h^2+4h+9} + 3)} = \frac{4}{6} = \frac{2}{3} \text{ ft/min}$$

Let $f(x) = \begin{cases} x^6 + x + 3 & \text{if } x < -1 \\ 5x^4 - 3x & \text{if } -1 < x < 1. \\ x^3 + x & \text{if } x > 1 \end{cases}$ **SCORE: * / 14 POINTS**

Find all discontinuities of $f(x)$ and find the type of each discontinuity (removable, jump or infinite). **DO NOT USE A GRAPH.**

$\lim_{x \to -1^-} f(x) = \lim_{x \to -1^-} (x^6 + x + 3) = 3$

$\lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} (5x^4 - 3x) = 2$

ONE SIDED LIMITS BOTH EXIST BUT ARE NOT EQUAL

$x = -1$ IS A JUMP DISCONT

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (5x^4 - 3x) = 2$

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (x^3 + x) = 2$

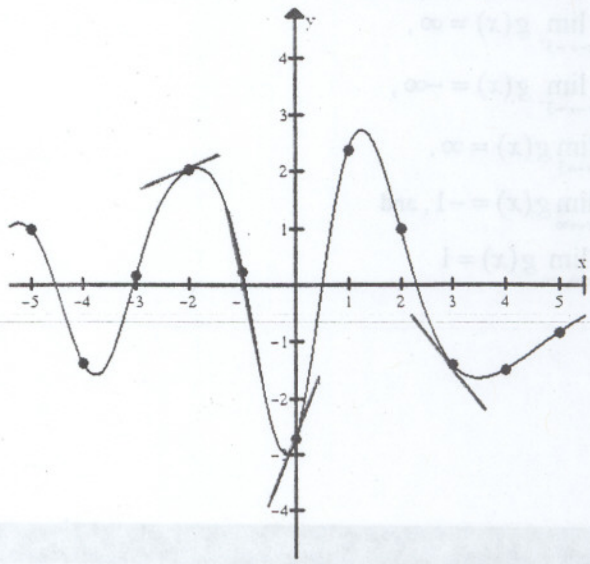
$\lim_{x \to 1} f(x)$ EXISTS BUT $f(1)$ DNE

$x = 1$ IS A REMOVABLE DISCONT

The graph of f is shown to the right.
Arrange the following from least (most negative) to greatest (most positive).

SCORE: ___ / 12 POINTS

$f'(-2)$ $f'(-1)$ $f'(0)$ $f'(3)$



$f(-1) < f(3) < f(-2) < f(0)$

LEAST … GREATEST

For the limit $\lim_{x\to -3}(x^2-3x-5)=13$, find a value of δ for ε. **Specifically, show the scratch work to find the value of δ in the proof of the limit. YOU DO NOT NEED TO WRITE A COMPLETE PROOF OF THE LIMIT.**

SCORE: ___ / 12 POINTS

IF $0<|x--3|<\delta$ THEN $|x^2-3x-5-13|<\varepsilon$

$0<|x+3|<\delta$ $|x^2-3x-18|<\varepsilon$

IF $\delta\le 1$ $|x-6||x+3|<\varepsilon$

$|x+3|<1$ $10|x+3|<\varepsilon$

$-1<x+3<1$ $|x+3|<\frac{\varepsilon}{10}$

$-10<x-6<-8$

$|x-6|<10$ $\delta = \text{MIN}\left(1,\frac{\varepsilon}{10}\right)$

Find the value of a if $\lim_{x\to -1}\frac{x^5-ax^3+2}{x^2+1}=-3$.

SCORE: ___ / 9 POINTS

$$\lim_{x\to -1}\frac{x^5-ax^3+2}{x^2+1}=\frac{-1-a(-1)+2}{(-1)^2+1}=\frac{1+a}{2}=-3$$

$1+a=-6$

$a=-7$

Sketch the graph of a function $f(x)$ which satisfies the following conditions: SCORE: _____ / 12 POINTS

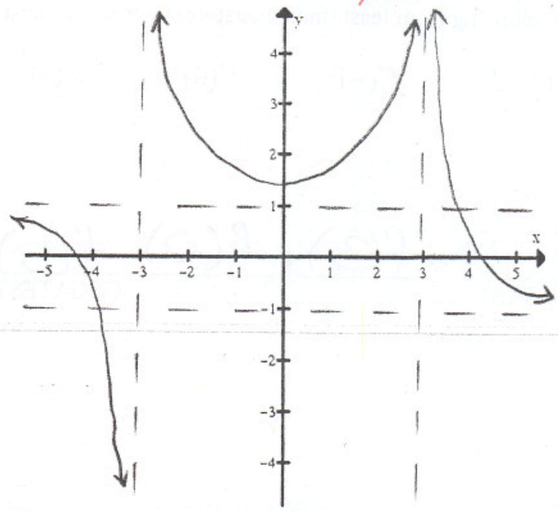
$\lim_{x\to -3^+} g(x) = \infty$,

$\lim_{x\to -3^-} g(x) = -\infty$,

$\lim_{x\to 3} g(x) = \infty$,

$\lim_{x\to \infty} g(x) = -1$, and

$\lim_{x\to -\infty} g(x) = 1$



Find all horizontal and vertical asymptotes of $f(x) = \dfrac{\sqrt{2x^6 - 7x^2 + 1}}{8 - x^3}$. Also, find both one-sided limits for each vertical asymptote. SCORE: _____ / 16 POINTS

V.A. $8 - x^3 = 0 \Rightarrow x = 2$ $\quad \lim_{x\to 2^+} f(x) = -\infty$ $\quad \lim_{x\to 2^-} f(x) = \infty$

$\frac{+}{0^-}$ $\quad \frac{+}{0^+}$

H.A. $\lim_{x\to\infty} \dfrac{\sqrt{2x^6 - 7x^2 + 1}}{8 - x^3} \cdot \dfrac{\frac{1}{x^3}}{\frac{1}{x^3}} = \lim_{x\to\infty} \dfrac{\sqrt{2 - \frac{7}{x^4} + \frac{1}{x^6}}}{\frac{8}{x^3} - 1} = \dfrac{\sqrt{2 - 0 + 0}}{0 - 1} = \sqrt{2}$

$\lim_{x\to -\infty} \dfrac{\sqrt{2x^6 - 7x^2 + 1}}{8 - x^3} \cdot \dfrac{\frac{1}{x^3}}{\frac{1}{x^3}} = \lim_{x\to -\infty} \dfrac{-\sqrt{2 - \frac{7}{x^4} + \frac{1}{x^6}}}{\frac{8}{x^3} - 1} = \dfrac{-\sqrt{2 - 0 + 0}}{0 - 1} = \sqrt{2}$

V.A. $x = 2$ $\quad$ H.A. $y = \pm\sqrt{2}$

The position of an object (in meters) at time t seconds, is given by the function $f(t) = t^3 - 5t + 2$. SCORE: _____ / 10 POINTS

Find the average velocity of the object over the interval $[1, 3]$. Specify the units.

$\dfrac{f(3) - f(1)}{3 - 1} = \dfrac{14 - (-2)}{2} = 8 \text{ m/s}$