

* = 100 POINTS
> 70%

What month is your birthday ?

What are the first 2 digits of your address ?

What are the last 2 digits of your zip code ?

What are the last 2 digits of your social security number ?

[IF YOU DO NOT HAVE A SOCIAL SECURITY NUMBER,
USE YOUR STUDENT ID NUMBER]

★ NO CALCULATORS ALLOWED
★ YOU MUST SHOW PROPER CALCULUS-LEVEL WORK AND LOGIC

State the definition of a vertical asymptote.

SCORE: * / 5 POINTS

f HAS A VERTICAL ASYMPTOTE AT $x=a$ IF

$$\lim_{x \rightarrow a^+} f(x) = \infty \text{ OR } -\infty, \text{ OR } \lim_{x \rightarrow a^-} f(x) = \infty \text{ OR } -\infty$$

State the Intermediate Value Theorem.

SCORE: * / 5 POINTS

IF f IS CONT. ON $[a,b]$ AND d IS BETWEEN $f(a)$ AND $f(b)$
THEN THERE IS A $c \in (a,b)$ SUCH THAT $f(c)=d$

State the definition of continuity at a point.

SCORE: * / 5 POINTS

f IS CONT. AT $x=a$ IF $f(a)$ EXISTS, $\lim_{x \rightarrow a} f(x)$ EXISTS
AND $\lim_{x \rightarrow a} f(x) = f(a)$

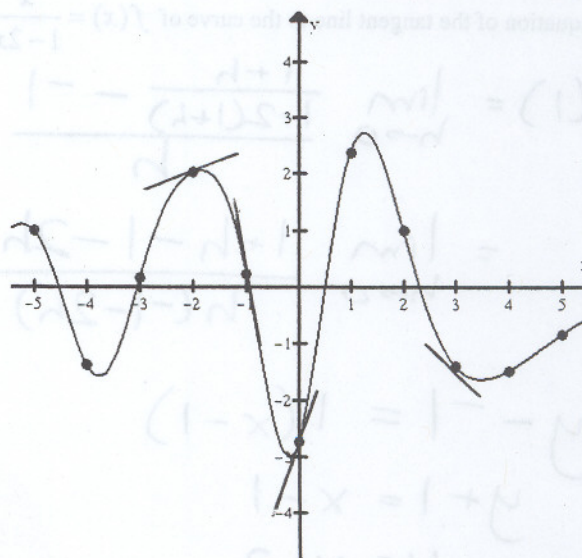
The graph of f is shown to the right.

SCORE: * / 12 POINTS

Arrange the following from least (most negative) to greatest (most positive).

$$f'(-2) \quad f'(-1) \quad f'(0) \quad f'(3)$$

$$\frac{f(-1)}{\text{LEAST}} < \frac{f(3)}{\text{GREATEST}} < \frac{f(-2)}{\text{GREATEST}} < \frac{f(0)}{\text{GREATEST}}$$



For the limit $\lim_{x \rightarrow -3} (x^2 - 3x - 5) = 13$, find a value of δ for ϵ . Specifically, show the scratch work

SCORE: ____ / 12 POINTS

to find the value of δ in the proof of the limit. YOU DO NOT NEED TO WRITE A COMPLETE PROOF OF THE LIMIT.

$$\text{IF } 0 < |x - (-3)| < \delta \text{ THEN } |x^2 - 3x - 5 - 13| < \epsilon$$

$$0 < |x + 3| < \delta \quad |x^2 - 3x - 18| < \epsilon$$

$$\text{IF } \delta \leq 1 \quad |x - 6||x + 3| < \epsilon$$

$$|x + 3| < 1 \quad 10|x + 3| < \epsilon$$

$$-1 < x + 3 < 1 \quad |x + 3| < \frac{\epsilon}{10}$$

$$-10 < x - 6 < -8$$

$$|x - 6| < 10 \quad \delta = \min(1, \frac{\epsilon}{10})$$

Find the value of a if $\lim_{x \rightarrow -1} \frac{x^5 - ax^3 + 2}{x^2 + 1} = -3$.

SCORE: * / 9 POINTS

$$\lim_{x \rightarrow -1} \frac{x^5 - ax^3 + 2}{x^2 + 1} = \frac{-1 - a(-1) + 2}{(-1)^2 + 1} = \frac{1 + a}{2} = -3$$

$$1 + a = -6$$

$$a = -7$$

Find the equation of the tangent line to the curve of $f(x) = \frac{x}{1-2x}$ at $x = 1$.

SCORE: * / 14 POINTS

$$f'(1) = \lim_{h \rightarrow 0} \frac{\frac{1+h}{1-2(1+h)} - (-1)}{h} = \lim_{h \rightarrow 0} \frac{\frac{1+h}{-1-2h} + 1}{h}$$

$$= \lim_{h \rightarrow 0} \frac{1+h-1-2h}{h(-1-2h)} = \lim_{h \rightarrow 0} \frac{-h}{h(-1-2h)} = 1$$

$$y - (-1) = 1(x - 1)$$

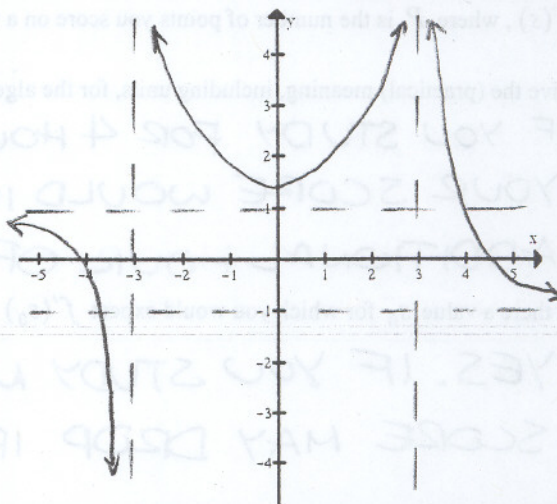
$$y + 1 = x - 1$$

$$y = x - 2$$

Sketch the graph of a function $f(x)$ which satisfies the following conditions:

SCORE: ~~8~~ / 12 POINTS

$$\begin{aligned}\lim_{x \rightarrow -3^+} g(x) &= \infty, \\ \lim_{x \rightarrow -3^-} g(x) &= -\infty, \\ \lim_{x \rightarrow 3} g(x) &= \infty, \\ \lim_{x \rightarrow \infty} g(x) &= -1, \text{ and} \\ \lim_{x \rightarrow -\infty} g(x) &= 1\end{aligned}$$



Find all horizontal and vertical asymptotes of $f(x) = \frac{\sqrt{2x^6 - 7x^2 + 1}}{8 - x^3}$. Also, find both one-sided limits for each vertical asymptote.

SCORE: ____ / 16 POINTS

V.A: $8 - x^3 = 0 \Rightarrow x = 2$ $\lim_{x \rightarrow 2^+} f(x) = -\infty$ $\lim_{x \rightarrow 2^-} f(x) = \infty$

H.A. $\lim_{x \rightarrow \infty} \frac{\sqrt{2x^6 - 7x^2 + 1}}{8 - x^3} \cdot \frac{\frac{1}{x^3}}{\frac{1}{x^3}} = \lim_{x \rightarrow \infty} \frac{\sqrt{2 - \frac{7}{x^4} + \frac{1}{x^6}}}{\frac{8}{x^3} - 1} = \frac{\sqrt{2 - 0 + 0}}{0 - 1} = -\sqrt{2}$

$\lim_{x \rightarrow -\infty} \frac{\sqrt{2x^6 - 7x^2 + 1}}{8 - x^3} \cdot \frac{\frac{1}{x^3}}{\frac{1}{x^3}} = \lim_{x \rightarrow -\infty} \frac{\sqrt{2 - \frac{7}{x^4} + \frac{1}{x^6}}}{\frac{8}{x^3} - 1} = \frac{-\sqrt{2 - 0 + 0}}{0 - 1} = \sqrt{2}$

V.A. $x = 2$ H.A. $y = \pm \sqrt{2}$

The position of an object (in meters) at time t seconds, is given by the function $f(t) = t^3 - 5t + 2$.

SCORE: ~~8~~ / 10 POINTS

Find the average velocity of the object over the interval $[1, 3]$. Specify the units.

$$\frac{f(3) - f(1)}{3 - 1} = \frac{14 - (-2)}{2} = 8 \text{ m/s}$$

Your score on a test depends on how much time you spent studying for it the 24 hours just before the test. SCORE: ____ / 12 POINTS
 Let $P = f(s)$, where P is the number of points you score on a test, and s is the number of hours you studied the day before.

[a] Give the (practical) meaning, including units, for the algebraic statement $f'(4) = 7$.

IF YOU STUDY FOR 4 HOURS THE DAY BEFORE A TEST,
 YOUR SCORE WOULD INCREASE 7 POINTS PER
 ADDITIONAL HOUR OF STUDYING.

[b] Is there a value s_0 for which you would expect $f'(s_0) < 0$? Why or why not?

YES. IF YOU STUDY MORE AND SLEEP LESS, YOUR
 SCORE MAY DROP IF $s_0 \approx 24$.

The position of an object (in feet) at time t minutes, is given by the function $f(t) = \sqrt{t^2 + 5}$.

SCORE:  / 14 POINTS

Find the instantaneous velocity of the object at time $t = 2$. Specify the units.

$$\begin{aligned} f'(2) &= \lim_{x \rightarrow 2} \frac{\sqrt{x^2 + 5} - 3}{x - 2} \cdot \frac{\sqrt{x^2 + 5} + 3}{\sqrt{x^2 + 5} + 3} = \lim_{x \rightarrow 2} \frac{x^2 + 5 - 9}{(x - 2)(\sqrt{x^2 + 5} + 3)} \\ &= \lim_{x \rightarrow 2} \frac{x^2 - 4}{(x - 2)(\sqrt{x^2 + 5} + 3)} = \frac{4}{6} = \frac{2}{3} \text{ ft/min} \end{aligned}$$

Let $f(x) = \begin{cases} x^6 + x + 3 & \text{if } x < -1 \\ 5x^4 - 3x & \text{if } -1 < x < 1 \\ x^3 + x & \text{if } x > 1 \end{cases}$

SCORE:  / 14 POINTS

Find all discontinuities of $f(x)$ and find the type of each discontinuity (removable, jump or infinite). DO NOT USE A GRAPH.

$$\lim_{x \rightarrow -1^-} f(x) = \lim_{x \rightarrow -1^-} (x^6 + x + 3) = 3$$

$$\lim_{x \rightarrow -1^+} f(x) = \lim_{x \rightarrow -1^+} (5x^4 - 3x) = 8$$

ONE SIDED LIMITS BOTH
 EXIST, BUT ARE NOT EQUAL

$x = -1$ IS A JUMP DISCONT.

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} (5x^4 - 3x) = 2$$

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} (x^3 + x) = 2$$

$\lim_{x \rightarrow 1} f(x)$ EXISTS BUT
 $f(1)$ DNE

$x = 1$ IS A REMOVABLE
 DISCONT