Math 1A Midterm 1 Review

1. THIS MIDTERM WILL COVER SECTIONS 2.1 THROUGH 2.7. SECTION 2.8 WILL BE PART OF MIDTERM 2.
2. IF YOU HAVE TAKEN DIFFERENTIAL CALCULUS BEFORE, DO NOT USE DIFFERENTIATION SHORTCUTS.
3. YOU SHOULD ONLY REQUIRE A CALCULATOR FOR QUESTIONS MARKED [C].
4. UNLESS A GRAPH IS GIVEN, YOU MUST BE ABLE TO SOLVE EACH PROBLEM WITHOUT A GRAPH.

[1][C] Estimate the slope of the tangent line to the curve $y=\sqrt{x+\sqrt{\cos x}}$ at the point $(0,1)$ using the slopes of several secant lines.

[2] The position of an object (in meters) at time t seconds, is given by the function $f(t)=t^2\cos\pi t$. Find the average velocity of the object over the interval $[1,5]$. Specify the units.

[3] Sketch the graph of a function $f(x)$ which satisfies the following conditions:

$\lim_{x\to -2^+} g(x)=-3$, $\lim_{x\to -2^-} g(x)=\infty$, $\lim_{x\to 1} g(x)=-\infty$, $\lim_{x\to -\infty} g(x)=2$, and $\lim_{x\to \infty} g(x)=-2$

[4] Prove that $\lim_{x\to 0} x\cos\frac{1}{x^2}=0$.

[5] Let $f(x)=\begin{cases} 2x-3 & \text{if } x<-1 \\ x^2-6 & \text{if } -1<x<2 \\ 4x-6 & \text{if } x\ge 2 \end{cases}$.

[a] Find $\lim_{x\to -2} f(x)$.

[b] Find $\lim_{x\to -1} f(x)$.

[c] Find $\lim_{x\to 2} f(x)$.

[6] Find the value of a if $\lim_{x\to 3}\frac{\sqrt{x^2+a}+10}{x+3}=2$.

[7] If $\lim_{x\to 2} f(x)=-3$ and $\lim_{x\to 2} g(x)=4$, find $\lim_{x\to 2}\frac{x^2 g(x)}{1+f(x)}$. Show clearly how the limit laws are used in your solution.

[8] Consider the limit $\lim_{x\to -4}(3x-1)=-13$.

[a] Find a value of δ for $\varepsilon=0.01$.

[b] Find a value of δ for ε (in general).

[9] Consider the limit $\lim_{x\to 3}(x^2-10x)=-21$. Find a value of δ for ε (in general).

[10] Find the discontinuities of $f(x)=\frac{x+2}{x^2-9}$, and find the one-sided limits at each discontinuity.

[11] Let $f(x)=\begin{cases} 2x+a & \text{if } x<-1 \\ 3-x & \text{if } -1<x<2 \\ bx-1 & \text{if } x\ge 2 \end{cases}$.

[a] Find the value of a so that $f(x)$ is continuous at $x=-1$.

[b] Find the value of b so that $f(x)$ is continuous at $x=2$.

[c] If $a=6$ and $b=3$, find all discontinuities of $f(x)$ and find the type of each discontinuity (removable, jump or infinite).

[12] Use the Intermediate Value Theorem to prove that the equation $\cos 2x = x^2$ has a solution in the interval $[0, \pi]$.

[13] Find all horizontal and vertical asymptotes of $f(x) = \dfrac{\sqrt{4+9x^2}}{2x-1}$.

[14] If $f(x) = x^3 - 3x + 2$, find $f'(-2)$ using both definitions of $f'(a)$.

[15] Find a function f and a number a such that the derivative of f at a is given by

[a] $\lim\limits_{h \to 0} \dfrac{\cos(\pi(h-1))+1}{h}$

[b] $\lim\limits_{x \to -2} \dfrac{x^2 - x - 6}{x+2}$

[16] The position of an object (in feet) at time t minutes, is given by the function $f(t) = \sqrt{t^2 - 5}$. Find the instantaneous velocity of the object at time $t = 3$. Specify the units.

[17] Find the equation of the tangent line to the curve of $f(x) = \dfrac{2x}{1-x}$ at $x = 2$.

[18] A baseball hits a parked car. The cost of repairing the damage depends on the impact velocity of the baseball (ie. the velocity the baseball was travelling when it hit the car). Let $C = f(v)$, where C is the repair cost in dollars ($), and v is the impact velocity in feet per second (ft/s).

[a] Give the (practical) meaning, including units, for the algebraic statement $f(20) = 30$.

[b] Give the (practical) meaning, including units, for the algebraic statement $f'(20) = 30$.

[c] Is there a value v_0 for which you would expect $f'(v_0) < 0$? Why or why not ?

[19] The number of iPhone applications sold per day by a certain vendor depends on the price of each application. Let $A = f(p)$, where A is the number of applications sold per day, and p is the price per application in dollars ($).

[a] Give the (practical) meaning, including units, for the algebraic statement $f(10) = 15$.

[b] Give the (practical) meaning, including units, for the algebraic statement $f'(10) = -5$.

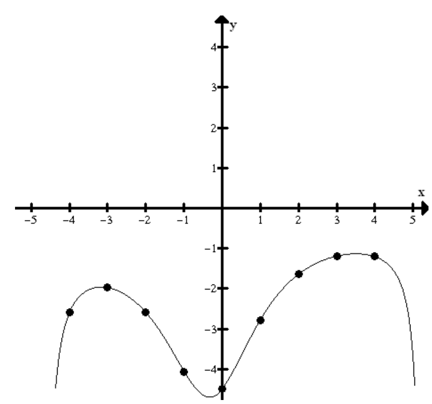
[c] Is there a value p_0 for which you would expect $f'(p_0) > 0$? Why or why not ?

[20] The graph of f is shown to the right. Arrange the following from least (most negative) to greatest (most positive).

$0 \qquad f'(-4) \qquad f'(-2) \qquad f'(2) \qquad f'(4)$

________ < ________ < ________ < ________ < ________

LEAST GREATEST



YOU MUST ALSO KNOW THE FOLLOWING DEFINITIONS AND THEOREMS:

Definition of limit, where the limit is a number
Definition of vertical asymptote
Definition of continuity at a point
Definition of removable discontinuity (from lecture)
Definition of jump discontinuity (from lecture)
Definition of derivative at a point
Squeeze Theorem
Intermediate Value Theorem