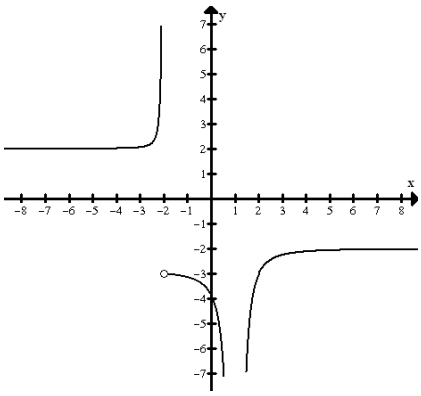
Math 1A Midterm 1 Review Answers

[1] $\frac{1}{2}$

[2] -6 meters per second

[3]



[4] Since $-1 \le \cos\frac{1}{x^2} \le 1$ for all x, therefore $-x \le x\cos\frac{1}{x^2} \le x$ for all x.

And since $\lim_{x\to 0}(-x) = \lim_{x\to 0} x = 0$, by the Squeeze Theorem, $\lim_{x\to 0} x\cos\frac{1}{x^2} = 0$ also.

[5] [a] -7 [b] -5 [c] DNE

[6] -5

[7] $\lim_{x\to 2}\frac{x^2 g(x)}{1+f(x)} = \frac{\lim_{x\to 2} x^2 g(x)}{\lim_{x\to 2}(1+f(x))} = \frac{\lim_{x\to 2} x \cdot \lim_{x\to 2} x \cdot \lim_{x\to 2} g(x)}{\lim_{x\to 2} 1 + \lim_{x\to 2} f(x)} = \frac{2\cdot 2\cdot 4}{1+(-3)} = -8$

[8] [a] $\delta = \frac{1}{300}$ [b] $\delta = \frac{\varepsilon}{3}$

[9] $\delta = \min(1, \frac{\varepsilon}{5})$ [OTHER ANSWERS POSSIBLE]

[10] discontinuities at $x = -3$ and $x = 3$

$\lim_{x\to -3^-} f(x) = -\infty$, $\lim_{x\to -3^+} f(x) = \infty$, $\lim_{x\to 3^-} f(x) = -\infty$, $\lim_{x\to 3^+} f(x) = \infty$

[11] [a] no such a [b] 1 [c] $x = -1$ removable, $x = 2$ jump

[12] Let $f(x) = \cos 2x - x^2$. Since $\cos 2x$ and x^2 are both continuous for all x, so is their difference $f(x) = \cos 2x - x^2$. Since $f(\pi) = 1 - \pi^2 < 0 < 1 = f(0)$, by the Intermediate Value Theorem, there is value c in the interval $(0, \pi)$ such that $f(c) = \cos 2c - c^2 = 0$, ie. $\cos 2c = c^2$. So the equation $\cos 2x = x^2$ has a solution in the interval $[0, \pi]$.

[13] $x = \frac{1}{2}$, $y = \pm\frac{3}{2}$

[14] $f'(-2) = \lim_{x\to -2}\frac{f(x)-f(-2)}{x-(-2)} = \lim_{x\to -2}\frac{x^3-3x+2}{x+2} = \lim_{x\to -2}\frac{(x+2)(x^2-2x+1)}{x+2} = \lim_{x\to -2}(x^2-2x+1) = 9$

$$f'(-2) = \lim_{h\to 0}\frac{f(-2+h)-f(-2)}{h} = \lim_{x\to -2}\frac{(-2+h)^3-3(-2+h)+2}{h} = \lim_{x\to -2}\frac{-8+12h-6h^2+h^3+6-3h+2}{h}$$

$$= \lim_{x\to -2}\frac{9h-6h^2+h^3}{h} = \lim_{x\to -2}(9-6h+h^2) = 9$$

[15] [a] $f(x)=\cos \pi x$, $a=-1$ [b] $f(x)=x^2-x$, $a=-2$

[16] 1.5 feet per minute

[17] $y+4=2(x-2)$

[18] [a] If the baseball hits the car at 20 ft/s, it will cost $\$30$ to repair the damage.

[b] If the baseball hits the car at 20 ft/s, it would cost $\$30$ more to repair the damage for each ft/s faster the baseball hit the car.

[c] No. The faster the baseball hits the car, the more it would cost to repair the damage.

[19] [a] If the price per application is $\$10$, the vendor sells 15 applications per day.

[b] If the price per application is $\$10$, the vendor will sell 5 fewer applications per day for each dollar he raises the price per application.

[c] Maybe, maybe not. In general, raising the price per application (making the application more expensive) should cause a decrease in the number of applications sold per day. However, if the price is set too low, people may not buy the application because they think it isn't a good product. In that case, raising the price per application might make the application look more valuable, and therefore result in more people buying it.

[20] $f'(-2) < f'(4) < 0 < f'(2) < f'(-4)$