Math 1A
Midterm 2 Review

The central focus of chapter 3 is how to find derivatives without using limits. To that end, you should be able to find any derivative from this chapter.

3.1	3-32
3.2	3-30
3.3	1-16
3.4	7-50
3.5	5-20, 25-30, 45-54
3.6	2-30, 37-50
3.REV	1-50

The question for section 3.9 will be one of the examples or assigned homework problems, but with different constants.

You should also be able to solve the following types of problems. Not knowing how to solve the following problems is a major reason why some students do worse on this midterm than they expect.

[1] The time required to defrost a piece of frozen meat in the refrigerator depends on the temperature inside the refrigerator. If $t = f(T)$, where t is the defrost time (in minutes), and T is the refrigerator temperature (in $^\circ C$), give the practical meaning (including units) of $f'(4) = -5$.

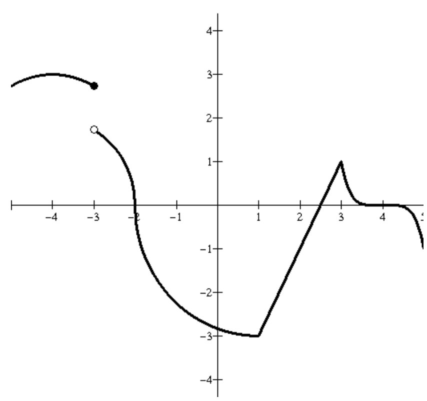
[2] **Using the definition of the derivative,** find the derivatives of the following functions.

[a] $f(t) = \dfrac{1}{\sqrt{1-t}}$ [b] $g(x) = \dfrac{4x}{2-x}$

[3] The graph of $f(x)$ is shown on the right.

[a] Find all x-coordinates where $f'(x)$ is undefined, and explain briefly why.

[b] Sketch a graph of $f'(x)$.



[4] The position of an object at time t is given by the function $s(t) = \dfrac{2t^3 + 4t^2 - 3}{\sqrt{t}}$ for $t > 0$.

[a] Find the velocity of the object at time $t = 1$.

[b] Find the acceleration function. **Simplify your answer.**

[5] Find the equations of the tangent lines to the curve $y = 1 + x^3$ that are perpendicular to $x + 12y = 1$.

[6] The line $y = 3x - 4$ is tangent to a quadratic function at the point $(1, -1)$. Find the equation of the tangent line to the quadratic function at $(2, 4)$.

[7] If $f(x) = \dfrac{x^3}{1+x^2}$, find $f''(1)$.

[8] The following table gives values and derivatives of two functions at various inputs.

x	–3	–2	–1	0	1	2	3	4
$f(x)$	–2	0	2	4	–3	–1	1	3
$f'(x)$	4	–1	–3	2	–4	3	–2	1
$g(x)$	–1	1	3	–3	4	–2	0	2
$g'(x)$	2	4	–4	–1	3	1	–3	–2

[a] If $k(x) = x^3 f(x)$, find the equation of the tangent line to $y = k(x)$ at $x = 2$.

[b] If $j(x) = \dfrac{x^2}{f(x)}$, find the equation of the tangent line to $y = j(x)$ at $x = -1$.

[c] If $m(x) = \tan^{-1}(g(x))$, find the equation of the tangent line to $y = m(x)$ at $x = -3$.

[d] If $n(x) = g(f(x))$, find the equation of the tangent line to $y = n(x)$ at $x = 4$.

[9] If $h(x) = f(x)g(x)$, find formulae for $h''(x)$ and $h'''(x)$. Based on your answers, guess a formula for $h^{(4)}(x)$ (the fourth derivative of $h(x)$.

[10] Find all x-coordinates in the interval $[0, 2\pi]$ where the tangent line to $f(x) = 4x - 3\tan x$ is horizontal.

[11] If $f(x) = xg(x^2)$, find a formula for $f''(x)$. Your answer may involve g, g' and/or g''.

[12] Find the equation of the tangent line to $(1 + x^2 y^3)^5 = x^4 e^y$ at $(-1, 0)$.

[13] Show that $y = ax^4$ and $x^2 + 4y^2 = b$ are orthogonal trajectories. **See section 3.5, questions 59-62.**

[14] If $y = (\sin x)^{\frac{1}{x}}$, find $\dfrac{dy}{dx}$.

[15] The limit $\lim\limits_{h \to 0} \dfrac{(h-1)e^{1-h} + e}{h}$ is the derivative of some function $f(x)$ at some point $x = a$. Find the function, the point, and the value of the limit.

Finally, you must know the following definitions, theorems and proofs.

Definitions
- vertical asymptote
- horizontal asymptote
- derivative at a point
- derivative (as a function)
- e

Theorems
- Squeeze Theorem
- Intermediate Value Theorem
- Differentiability implies continuity

Proofs
- derivatives of $\sin x$, $\cos x$, $\tan x$, $\csc x$, $\sec x$ and $\cot x$
  - using the definition of the derivative, without using the derivatives of any other trigonometric function
  - you may use the limits $\lim\limits_{x \to 0} \dfrac{\sin x}{x} = 1$ and $\lim\limits_{x \to 0} \dfrac{\cos x - 1}{x} = 0$ without proving them
- derivatives of $\tan x$, $\csc x$, $\sec x$ and $\cot x$
  - using the quotient rule on the derivatives of $\sin x$ and $\cos x$
- derivatives of $\sin^{-1} x$, $\cos^{-1} x$, $\tan^{-1} x$, and $\ln x$
  - using implicit differentiation with the derivatives of $\sin x$, $\cos x$, $\tan x$ and e^x