Math 1A (9:30am – 10:20am)
Midterm 3
Wed Mar 17, 2010

SCORE: ___ / 132 POINTS + ___ / 8 POINTS

What month is your birthday ? ___ ___
What are the first 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___
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NO CALCULATORS ALLOWED ON THIS SECTION
SHOW PROPER CALCULUS-LEVEL WORK

State the definition of "local minimum". **SCORE: ___ / 4 POINTS**

f HAS A LOCAL MINIMUM AT c
IF $f(c) \le f(x)$
FOR ALL x IN AN OPEN INTERVAL AROUND c

State the definition of "critical number". **SCORE: ___ / 4 POINTS**

c IS A CRITICAL NUMBER OF f
IF c IS IN THE DOMAIN OF f
AND $f'(c)=0$ OR IS UNDEFINED

State the Mean Value Theorem. **SCORE: ___ / 4 POINTS**

IF f IS CONTINUOUS ON $[a,b]$ AND DIFFERENTIABLE ON (a,b)
THEN THERE IS A $c \in (a,b)$ SUCH THAT $f'(c) = \frac{f(b)-f(a)}{b-a}$

Evaluate the following limits. **SCORE: ___ / 18 POINTS**

[a] $\lim_{x\to 1} \frac{x^3-4x^2+5x-2}{2x^4-8x+4} = \frac{1-4+5-2}{2-8+4} = 0$

[b] $\lim_{x\to 0}(1-\sin 3x)^{\frac{2}{x}}$ 1^∞

$\lim_{x\to 0} \ln(1-\sin 3x)^{\frac{2}{x}}$

$= \lim_{x\to 0} \frac{2\ln(1-\sin 3x)}{x}$ $\frac{0}{0}$

$= \lim_{x\to 0} \frac{\frac{2}{1-\sin 3x}\cdot -3\cos 3x}{1}$

$= 2\cdot(-3)$

$= -6$

$\lim_{x\to 0}(1-\sin 3x)^{\frac{2}{x}} = e^{-6}$

Consider the following forms which might appear when trying to find a limit. **SCORE: ___ / 11 POINTS**

(1)	$\infty+\infty$	(2)	$\infty-\infty$	(3)	$\infty\cdot\infty$	(4)	$\infty\cdot 0$
(5)	$\frac{\infty}{\infty}$	(6)	$\frac{\infty}{0}$	(7)	$\frac{0}{\infty}$	(8)	$\frac{0}{0}$
(9)	∞^{∞}	(10)	0^{∞}	(11)	1^{∞}	(12)	∞^{0}

[a] Which forms are indeterminate, and may require L'Hopital's Rule to find the limit ? 2, 4, 5, 8, 11, 12

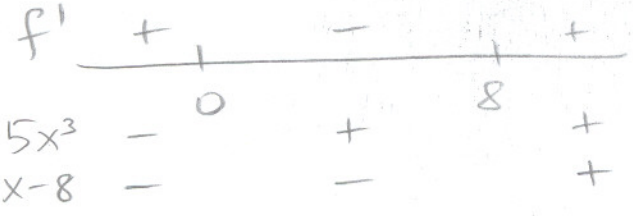
[b] Which forms do not require L'Hopital's Rule, and automatically give a limit of ∞ ? 1, 3, 9

[c] Which forms do not require L'Hopital's Rule, and automatically give a limit of 0 ? 7, 10

Let $f(x)=x^5-10x^4+5$. POLYNOMIAL $\Rightarrow$ ALL DERIVATIVES DEFINED **SCORE: ___ / 25 POINTS**

[a] Find all intervals over which $f(x)$ is increasing.

$f'(x)=5x^4-40x^3=5x^3(x-8)=0$ @ $x=0,8$



$5x^3$	−	+	+
$x-8$	−	−	+

$(-\infty, 0]$ AND $[8, \infty)$

[b] Find the critical numbers of $f(x)$, and explain what the ***second derivative test*** tells you about each one.

$x=0, 8$

$f''(x)=20x^3-120x^2=20x^2(x-6)$

$f''(0)=0$ NO CONCLUSION

$f''(8)=20\cdot 8^2(8-6)>0$ LOCAL MIN

CONCAVE UP ∪

[c] Find the inflection points of $f(x)$.

$f''(x)=0$ @ $x=0,6$

$20x^2$	+	+	+
$x-6$	−	−	+

$x=6$

Your group was working on graphing a function $f(x)$ and came up with the following analysis: **SCORE: ___ / 14 POINTS**

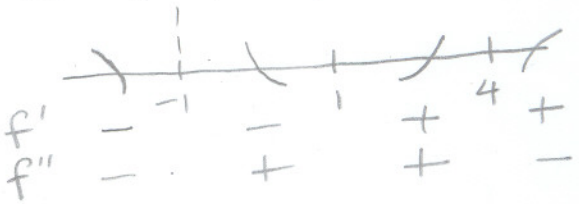
$f(0)=0,\ \lim_{x\to-\infty} f(x)=0,\ \lim_{x\to-1^-} f(x)=-\infty,\ \lim_{x\to-1^+} f(x)=\infty,$

$f'(x)<0$ on $(-\infty,-1)$ and $(-1,1)$, $f'(x)>0$ on $(1,\infty)$,

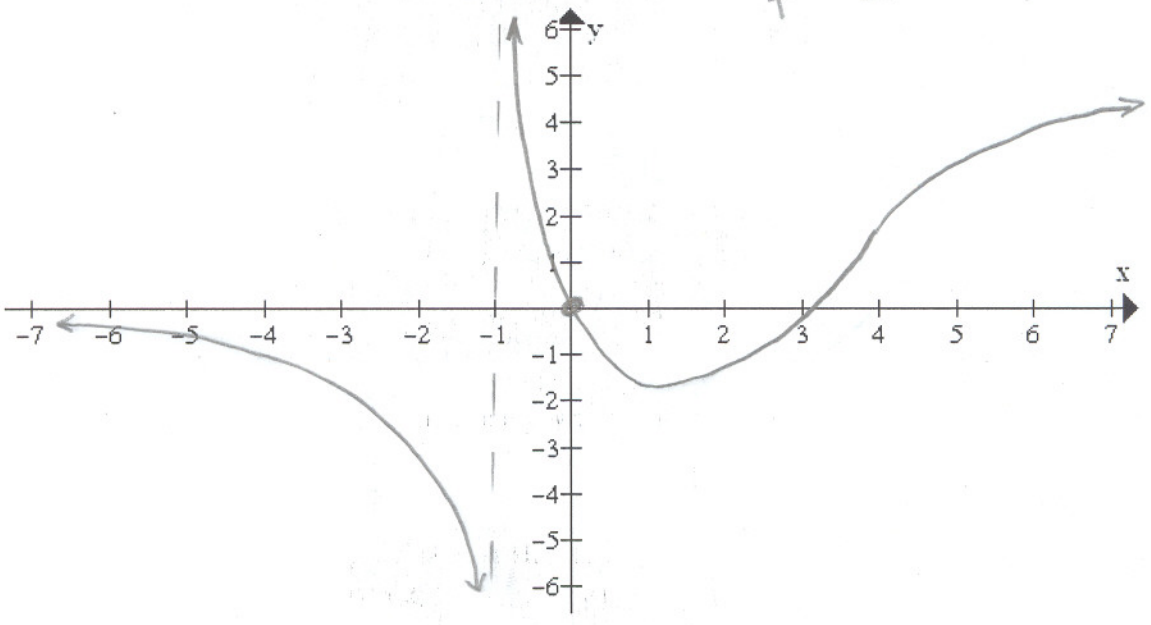
$f''(x)<0$ on $(-\infty,-1)$ and $(4,\infty)$, $f''(x)>0$ on $(-1,4)$

H.A. $y=0$ $(x\to-\infty)$

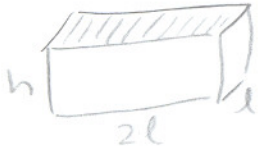
V.A. $x=-1$



Sketch a possible graph of $f(x)$.



A rectangular storage container with an open top is to have a volume of 24 cubic meters. The length of its base is twice the width. Material for the base costs \$9 per square meter. Material for the sides costs \$4 per square meter. Find the cost of materials for the cheapest such container. **SCORE: ___ / 14 POINTS**



MINIMIZE COST OF MATERIAL $C=9(2l^2)+4(2(lh)+2(2lh))$

$=18l^2+24lh$

$2l^2h=24 \Rightarrow h=\frac{12}{l^2}$

$C=18l^2+288l^{-1}$ $\quad l\in(0,\infty)$

$C'=36l-288l^{-2}$ UNDEFINED @ $l=0 \notin$ DOMAIN

$=0$ IF $36l=288l^{-2}$

$l^3=8$

$l=2$

$C''=36+576l^{-3}>0$ ON $(0,\infty)$

GLOBAL MIN

$C(2)=18(2)^2+288(\frac{1}{2})$

$=72+144$

$=\$216$

Find the absolute maxima and minima of $f(x) = (4x^2 - 1)^{\frac{2}{3}}$ on $[-1, 2]$. **SCORE: ___ / 14 POINTS**

$f'(x) = \frac{2}{3}(4x^2-1)^{-\frac{1}{3}}(8x)$ IS UNDEFINED @ $x = \pm\frac{1}{2} \in$ DOMAIN

$= 0$ @ $x = 0$

x	$f(x)$	
0	$(-1)^{2/3} = 1$	
$-\frac{1}{2}$	0	$\Rightarrow$ MIN
$\frac{1}{2}$	0	$\Leftarrow$ MIN
-1	$3^{2/3} = \sqrt[3]{9}$	
2	$15^{2/3} = \sqrt[3]{225}$	$\leftarrow$ MAX

Determine if Rolle's Theorem applies to $f(x) = \sqrt[3]{x^2 - 1}$ on $[-2, 2]$. If so, find the value of c guaranteed. If not, explain briefly why not. **SCORE: ___ / 10 POINTS**

$f'(x) = \frac{1}{3}(x^2-1)^{-\frac{2}{3}}(2x)$ IS UNDEFINED @ $x = \pm 1 \in$ INTERVAL

SO f' IS NOT DIFFERENTIABLE ON $[-2, 2]$

SO ROLLE'S TH'M DOES NOT APPLY

Use differentials or a linear approximation to estimate $\sin^{-1} 0.49$. Your final answer may involve e, π and/or radicals. **SCORE: ___ / 14 POINTS**

$f(x) = \sin^{-1} x$ @ $x = \frac{1}{2}$ $\quad f'(x) = \frac{1}{\sqrt{1-x^2}}$

$L(x) = f(\frac{1}{2}) + f'(\frac{1}{2})(x - \frac{1}{2})$

$= \sin^{-1}\frac{1}{2} + \frac{1}{\sqrt{1-(\frac{1}{2})^2}}(x - \frac{1}{2})$

$= \frac{\pi}{6} + \frac{2}{\sqrt{3}}(x - \frac{1}{2})$

$dy = \frac{1}{\sqrt{1-x^2}}\,dx$

$= \frac{1}{\sqrt{1-(\frac{1}{2})^2}}(0.49 - \frac{1}{2})$

$= \frac{2}{\sqrt{3}} \cdot \frac{-1}{100} = \frac{-1}{50\sqrt{3}}$

$f(0.49) \approx f(\frac{1}{2}) + dy$

$f(0.49) \approx L(0.49) = \frac{\pi}{6} + \frac{2}{\sqrt{3}}(0.49 - \frac{1}{2}) = \frac{\pi}{6} + \frac{2}{\sqrt{3}} \cdot \frac{-1}{100} = \frac{\pi}{6} - \frac{1}{50\sqrt{3}}$

TURN IN THIS SECTION IN ORDER TO RECEIVE YOUR CALCULATOR
YOU CANNOT GO BACK TO THIS SECTION AFTER YOU TURN IT IN

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CALCULATORS ALLOWED ON THIS SECTION

Use Newton's method to solve $\sin 2x = \cos x$ with $x_0 = 4$. **SCORE: ___ / 8 POINTS**

NOTE: The equation has multiple roots, and Newton's method will jump around before it settles down to a final answer.

[a] What should you type into your calculator ? $f(x) = \sin 2x - \cos x = 0$

$4 \rightarrow x$

$x - (\sin(2x) - \cos(x)) / (2\cos(2x) + \sin(x)) \rightarrow x$ ENTER ENTER...

[b] Fill in the blanks (round off to 6 decimal places).

$x_1 =$ 5.568045 $x_2 =$ 0.921938 FINAL ANSWER = 1.570796