Math 1A (9:30am – 10:20am)
Quiz 3 Version A
Fri Jan 22, 2009

SCORE: ___ / 20 POINTS

What month is your birthday ? ___ ___
What are the first 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___
What are the last 2 digits of your social security number ? ___ ___
[IF YOU DO NOT HAVE A SOCIAL SECURITY NUMBER, USE YOUR STUDENT ID NUMBER]

NO CALCULATORS ALLOWED

State the definition of continuity at a point. **SCORE: ___ / 2 POINTS**

State the Intermediate Value Theorem. **SCORE: ___ / 2 POINTS**

Find the values of a and b such that $f(x) = \begin{cases} a - 3x & \text{if } x < 4 \\ 7 & \text{if } x = 4 \\ 2x - b & \text{if } x > 4 \end{cases}$ is continuous at $x = 4$. **SCORE: ___ / 3 POINTS**

$\lim_{x \to 4^-} f(x) = \lim_{x \to 4^-} (a - 3x) = a - 12 = 7$ IF $a = 19$

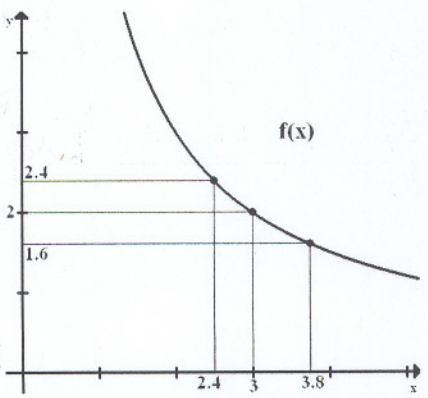
$\lim_{x \to 4^+} f(x) = \lim_{x \to 4^+} (2x - b) = 8 - b = 7$ IF $b = 1$

$\frac{1}{2}$ POINT EACH

Use the given graph of a function $f(x)$ to find a number δ such that **SCORE: ___ / 2 POINTS**

if $0 < |x - 3| < \delta$, then $|f(x) - 2| < 0.4$

$3 - 2.4 = 0.6$ $\frac{1}{2}$
$3.8 - 3 = 0.8$ $\frac{1}{2}$
$\delta = 0.6$ 1



CONTINUED ON OTHER SIDE ➡

Using the definition of continuity at a point, explain why the following functions are NOT continuous at the given points. Be as specific as possible. **DO NOT USE GRAPHS.** **SCORE: ___ / 6 POINTS**

[a] $f(x)=\begin{cases}\dfrac{1-\dfrac{4}{x^2}}{x+2} & \text{if } x\neq 2\\ 1 & \text{if } x=2\end{cases}$ at $x=2$

$\lim_{x\to 2} f(x) = \lim_{x\to 2} \dfrac{1-\frac{4}{x^2}}{x+2}$ 1

$= \frac{0}{4} = 0 \neq 1$ ½

$\lim_{x\to 2} f(x) \neq f(2)$ 1½

[b] $f(x)=\begin{cases}x^2-2 & \text{if } x<-1\\ 3x^{-1}-2 & \text{if } -1\le x<1\\ 2-x^3 & \text{if } x>1\end{cases}$ at $x=1$

$f(1)$ DNE 3

Is the following proof valid ? Explain briefly why or why not. **SCORE: ___ / 2 POINTS**

Prove that $f(x)=1-\dfrac{4}{x^2}=0$ for some $x\in(-1,3)$.

Proof: Since $f(-1)=1-\dfrac{4}{1}=-3$ and $f(3)=1-\dfrac{4}{9}=\dfrac{5}{9}$, and since $-3<0<\dfrac{5}{9}$,

by the Intermediate Value Theorem, $f(x)=1-\dfrac{4}{x^2}=0$ for some $x\in(-1,3)$.

½ IT IS NOT VALID. $f(x)$ IS NOT CONTINUOUS AT $0\in(-1,3)$ 1

SINCE $f(0)$ DNE. SO THE IVT DOES NOT APPLY. ½

For the limit $\lim_{x\to 2}(x^2-9x+4)=-10$, find a value of δ for ε. **Specifically, show the scratch work to find the value of δ in the proof of the limit. YOU DO NOT NEED TO WRITE A COMPLETE PROOF OF THE LIMIT.** **SCORE: ___ / 3 POINTS**

IF $0<|x-2|<\delta$, THEN $|x^2-9x+4-(-10)|<\varepsilon$

$|x^2-9x+14|<\varepsilon$

$|x-7||x-2|<\varepsilon$ ¼

IF $\delta\le 1$, ½

$|x-2|<\delta$

SO $-1<x-2<1$ ¼

$-6<x-7<-4$ ¼

$4<|x-7|<6$ ½

$6\delta<\varepsilon$ ½

$\delta=\dfrac{\varepsilon}{6}$ ¼

SO $\delta=$ MINIMUM$(1,\frac{\varepsilon}{6})$ ½