Math 1A (9:30am – 10:20am)
Quiz 3 Version A
Fri Jan 22, 2009

SCORE: ___ / 20 POINTS

What month is your birthday ? ___ ___
What are the first 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___
What are the last 2 digits of your social security number ? ___ ___
[IF YOU DO NOT HAVE A SOCIAL SECURITY NUMBER, USE YOUR STUDENT ID NUMBER]

NO CALCULATORS ALLOWED

State the Intermediate Value Theorem. **SCORE: ___ / 2 POINTS**

State the definition of continuity at a point. **SCORE: ___ / 2 POINTS**

Find the values of a and b such that $f(x) = \begin{cases} 3x - a & \text{if } x < 3 \\ 7 & \text{if } x = 3 \\ b - 2x & \text{if } x > 3 \end{cases}$ is continuous at $x = 3$. **SCORE: ___ / 3 POINTS**

$\lim_{x \to 3^-} f(x) = \underline{\lim_{x \to 3^-} (3x - a)} = \underline{9 - a = 7}$ IF $\underline{a = 2}$

$\frac{1}{2}$ POINT EACH

$\lim_{x \to 3^+} f(x) = \underline{\lim_{x \to 3^+} (b - 2x)} = \underline{b - 6 = 7}$ IF $\underline{b = 13}$

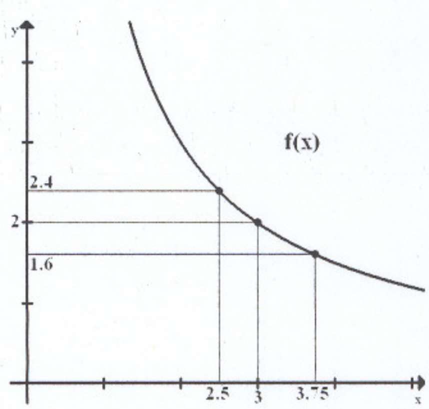
Use the given graph of a function $f(x)$ to find a number δ such that **SCORE: ___ / 2 POINTS**

if $0 < |x - 3| < \delta$, then $|f(x) - 2| < 0.4$

$3 - 2.5 = \underline{0.5}$ $\frac{1}{2}$

$3.75 - 3 = \underline{0.75}$ $\frac{1}{2}$

$\underline{\delta = 0.5}$ 1



CONTINUED ON OTHER SIDE ➜

Using the definition of continuity at a point, explain why the following functions are NOT continuous at the given points. Be as specific as possible. **DO NOT USE GRAPHS.** **SCORE: ___ / 6 POINTS**

[a] $f(x)=\begin{cases} 2x-1 & \text{if } x<2 \\ 3 & \text{if } x=2 \\ 5-2x & \text{if } x>2 \end{cases}$ at $x=2$

[b] $f(x)=\begin{cases} \dfrac{1-\frac{4}{x^2}}{x+2} & \text{if } x\neq 2 \\ 1 & \text{if } x=2 \end{cases}$ at $x=2$

$\lim\limits_{x\to 2^-} f(x) = \underbrace{\lim\limits_{x\to 2^-}(2x-1)}_{\frac{1}{2}} = \underbrace{3}_{\frac{1}{2}}$

$\lim\limits_{x\to 2^+} f(x) = \underbrace{\lim\limits_{x\to 2^+}(5-2x)}_{\frac{1}{2}} = \underbrace{1}_{\frac{1}{2}}$

$\underbrace{\lim\limits_{x\to 2} f(x) \text{ DNE}}_{1}$

$\underbrace{\lim\limits_{x\to 2} f(x)}_{1} = \lim\limits_{x\to 2} \dfrac{1-\frac{4}{x^2}}{x+2}$

$= \dfrac{0}{4} \underbrace{=0}_{\frac{1}{2}} \neq 1$

$\underbrace{\lim\limits_{x\to 2} f(x) \neq f(2)}_{1\frac{1}{2}}$

Is the following proof valid ? Explain briefly why or why not. **SCORE: ___ / 2 POINTS**

Prove that $f(x)=1-\dfrac{4}{x^2}=0$ for some $x\in(-1,3)$.

Proof: Since $f(-1)=1-\dfrac{4}{1}=-3$ and $f(3)=1-\dfrac{4}{9}=\dfrac{5}{9}$, and since $-3<0<\dfrac{5}{9}$,

by the Intermediate Value Theorem, $f(x)=1-\dfrac{4}{x^2}=0$ for some $x\in(-1,3)$.

$\frac{1}{2}$ IT IS NOT VALID. $f(x)$ IS DISCONTINUOUS AT $x=0\in(-1,3)$ 1

SINCE $f(0)$ DNE. SO THE IVT DOES NOT APPLY $\frac{1}{2}$

For the limit $\lim\limits_{x\to 3}(x^2-8x+5)=-10$, find a value of δ for ε. **Specifically, show the scratch work to find the value of δ in the proof of the limit. YOU DO NOT NEED TO WRITE A COMPLETE PROOF OF THE LIMIT.** **SCORE: ___ / 3 POINTS**

IF $0<|x-3|<\delta$, THEN $|x^2-8x+5-(-10)|<\varepsilon$

$|x^2-8x+15|<\varepsilon$

$\underbrace{|x-3||x-5|<\varepsilon}_{\frac{1}{4}}$

$\underbrace{\text{IF } \delta\leq 1}_{\frac{1}{2}}$

$|x-3|<1$

SO $\underbrace{-1<x-3<1}_{\frac{1}{4}}$

$\underbrace{-3<x-5<-1}_{\frac{1}{4}}$

$\underbrace{1<|x-5|<3}_{\frac{1}{2}}$

$\underbrace{3\delta<\varepsilon}_{\frac{1}{2}}$

$\underbrace{\delta=\frac{\varepsilon}{3}}_{\frac{1}{4}}$

SO $\underbrace{\delta=\text{MINIMUM}\left(1,\frac{\varepsilon}{3}\right)}_{\frac{1}{2}}$