

SCORE: ____ / 20 POINTS

What month is your birthday? _____

What are the first 2 digits of your address? _____

What are the last 2 digits of your zip code? _____

What are the last 2 digits of your social security number? _____

[IF YOU DO NOT HAVE A SOCIAL SECURITY NUMBER,
USE YOUR STUDENT ID NUMBER]**NO CALCULATORS ALLOWED**

The lifespan of a light bulb depends on how many hours a day you use it. Let $L = f(u)$, where L is the lifespan (in months), and u is the daily usage of the lightbulb (in hours). Give the practical meaning, including units, for the statement $f'(5) = -1$. SCORE: ____ / 2 POINTS

IF A LIGHTBULB IS USED 5 HOURS A DAY,
IT WILL LAST 1 MONTH LESS FOR EACH ADDITIONAL
HOUR YOU USE IT DAILY.

State the Intermediate Value Theorem.

SCORE: ____ / 1 POINTS

SEE YOUR TEXTBOOK / NOTES

Let $f(x) = \frac{x^2 - 8x + 6}{1 - 2x}$.

SCORE: ____ / 5 POINTS

[a] Find $f'(x)$. SIMPLIFY YOUR ANSWER.

$$f'(x) = \frac{(2x - 8)(1 - 2x) - (x^2 - 8x + 6)(-2)}{(1 - 2x)^2}$$

$$= \frac{2x - 4x^2 - 8 + 16x + 2x^2 - 16x + 12}{(1 - 2x)^2}$$

$$= \frac{-2x^2 + 2x + 4}{(1 - 2x)^2}$$

[b] Are there any points on the curve $y = f(x)$ where the tangent line is horizontal? If so, find them. If not, explain briefly why not.

$f'(x) = 0$ IF $-2x^2 + 2x + 4 = 0$ AND $(1 - 2x)^2 \neq 0$

$$x^2 - x - 2 = 0$$

$$(x - 2)(x + 1) = 0$$

$x = 2, -1$

Using the definition of the derivative, find the derivative of $f(x) = \frac{x}{1+2x}$.

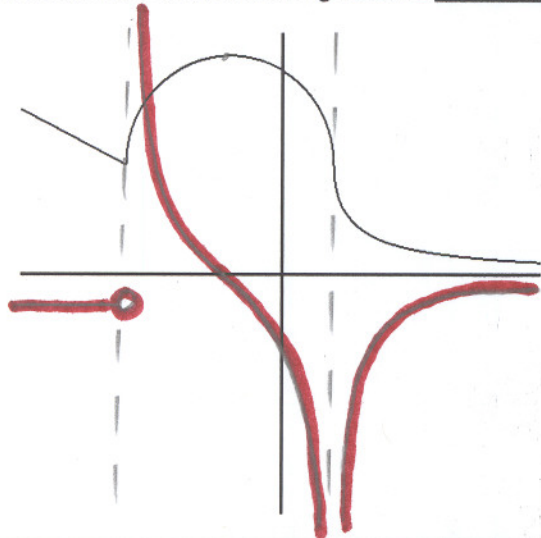
SCORE: ___ / 3 POINTS

$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} \frac{\frac{x+h}{1+2(x+h)} - \frac{x}{1+2x}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{(x+h)(1+2x) - x(1+2x+2h)}{h(1+2x+2h)(1+2x)} \\
 &= \lim_{h \rightarrow 0} \frac{x + 2x^2 + h + 2xh - x - 2x^2 - 2xh}{h(1+2x+2h)(1+2x)} \\
 &= \lim_{h \rightarrow 0} \frac{1}{(1+2x+2h)(1+2x)} = \frac{1}{(1+2x)^2}
 \end{aligned}$$

$-\frac{1}{2}$ IF YOU FORGOT
"lim" AT ANY
 $h \rightarrow 0$
POINT

Sketch the derivative of the following function on the same set of axes.

SCORE: ___ / 3 POINTS



Find the requested derivatives.

SCORE: ___ / 6 POINTS

[a] If $f(t) = \frac{(t+2)^2}{\sqrt{t}}$, find $f''(t)$.

$$\begin{aligned}
 f(t) &= (t^2 + 4t + 4)t^{-\frac{1}{2}} \\
 &= t^{\frac{3}{2}} + 4t^{\frac{1}{2}} + 4t^{-\frac{1}{2}} \\
 f'(t) &= \frac{3}{2}t^{\frac{1}{2}} + 2t^{-\frac{1}{2}} - 2t^{-\frac{3}{2}} \\
 f''(t) &= \frac{3}{4}t^{-\frac{1}{2}} - t^{-\frac{3}{2}} + 3t^{-\frac{5}{2}}
 \end{aligned}$$

[b] If $f(x) = (2x^4 - 5x^3 + e^2)3^x$, find $f'(0)$.

$$\begin{aligned}
 f'(x) &= (8x^3 - 15x^2)3^x + (2x^4 - 5x^3 + e^2)\ln(3)3^x \\
 f'(0) &= 0 + e^2 \ln(3) \\
 &= e^2 \ln(3)
 \end{aligned}$$